#6. $\vec{F} = ye^x\,\vec{i} + (e^x + e^y)\,\vec{j}$

$$\operatorname{curl}\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ ye^x & e^x + e^y & 0 \end{vmatrix}$$

$$= 0\,\vec{i} + 0\,\vec{j} + \left(\frac{\partial}{\partial x}(e^x + e^y) - \frac{\partial}{\partial y}(ye^x)\right)\vec{k}$$

$$= (e^x - e^x)\,\vec{k}$$

$$= 0\,\vec{k}$$

$$= \vec{0}$$

$\therefore$ $\vec{F}$ is conservative

(i) $\frac{\partial f}{\partial x} = ye^x$

(ii) $\frac{\partial f}{\partial y} = e^x + e^y$

$$f(x,y) = \int \frac{\partial f}{\partial x}\, dx \overset{(i)}{=} \int y e^x\, dx$$

$$= y e^x + g(y)$$

$$\Rightarrow \frac{\partial f}{\partial y} = e^x + g'(y) \overset{(ii)}{=} e^x + e^y$$

$$\Rightarrow g'(y) = e^y$$

$$\Rightarrow g(y) = \int e^y dy = e^y + c$$

$$\therefore f(x,y) = y e^x + e^y + c.$$

#16. $\vec{F} = (y^2z + 2xz^2)\vec{i} + 2xyz\,\vec{j} + (xy^2 + 2x^2z)\vec{k}$

(i) $\frac{\partial f}{\partial x} = y^2z + 2xz^2$

(ii) $\frac{\partial f}{\partial y} = 2xyz$

(iii) $\frac{\partial f}{\partial z} = xy^2 + 2x^2z$

$$f(x,y,z) = \int \frac{\partial f}{\partial x}\,dx \overset{(i)}{=} \int y^2z + 2xz^2\,dx$$

$$= xy^2z + x^2z^2 + g(y,z)$$

$$\Longrightarrow \frac{\partial f}{\partial y} = 2xyz + 0 + \frac{\partial g}{\partial y} \overset{(ii)}{=} 2xyz$$

$$\Longrightarrow \frac{\partial g}{\partial y} = 0$$

$$\Longrightarrow g(y,z) = \int \frac{\partial g}{\partial y}\,dy = h(z)$$

$$\therefore f(x,y,z) = xy^2z + x^2z^2 + h(z)$$

$$\Longrightarrow \frac{\partial f}{\partial z} = xy^2 + 2x^2z + h'(z) \overset{(iii)}{=} xy^2 + 2x^2z$$

$$\Longrightarrow h'(z) = 0$$

$$\Longrightarrow h(z) = C \text{ for some constant } C.$$

$$\therefore f(x,y,z) = xy^2z + x^2z^2 + C.$$

(b) $$\int_C \vec{F} \cdot d\vec{r} = \int_C \nabla f \cdot d\vec{r}$$

$$= f(\vec{r}(1)) - f(\vec{r}(0))$$

$$= \cancel{f(\sqrt{0},\ 0+1,\ 0^2)}$$

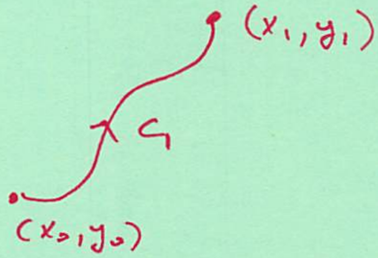
$$= f(\sqrt{1},\ 1+1,\ 1^2) - f(\sqrt{0},\ 0+1,\ 0^2)$$

$$= f(1,2,1) - f(0,1,0)$$

$$= 1 \cdot 2^2 \cdot 1 + 1^2 \cdot 1^2 - (0 + 0)$$

$$= 5$$

(20a) $0 = \int_{C_1} \vec{F} \cdot d\vec{r}$



$$= \sin(x_1 - 2y_1) - \sin(x_0 - 2y_0)$$

fundamental theorem.

Pick $(x_0, y_0) = (0, 0)$ and $(x_1, y_1) = (2\pi, 0)$

$\Longrightarrow$ $x_0 - 2y_0 = 0$ and $x_1 - 2y_1 = 2\pi$

$$0 = \sin(2\pi) - \sin(0)$$

$$= 0 - 0$$

$$= 0.$$

Hence, the straight line path from $(0,0)$ to $(2\pi, 0)$ is not closed and satisfies

$$\int_{C_1} \vec{F} \cdot d\vec{r} = 0.$$