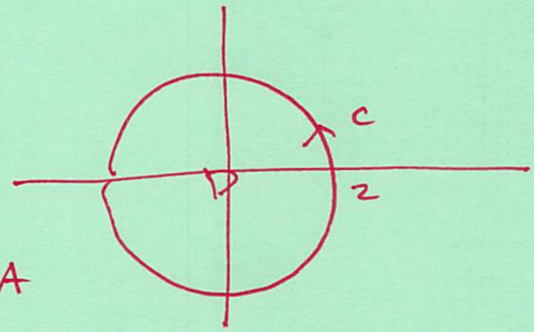
#2. $\oint_C y\,dx - x\,dy$



$$= \iint_D \frac{\partial}{\partial x}(-x) - \frac{\partial}{\partial y}(y)\,dA$$

$$= \iint_D -1 - 1\,dA$$

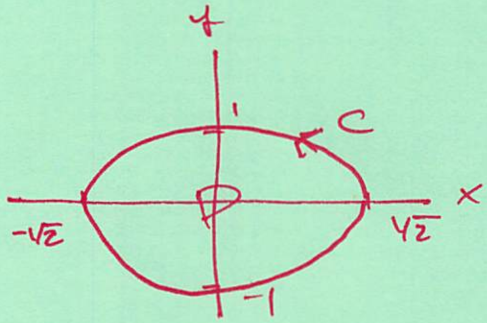
$$= -2 \cdot \iint_D 1\,dA$$

$$= -2 \cdot A(D)$$

$$= -2 \cdot \pi(2)^2$$

$$= -8\pi$$

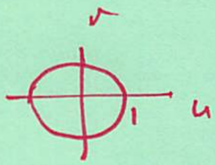
$$\# 8. \quad \int_C y^4\,dx + 2xy^3\,dy$$



$$= \iint_D \frac{\partial}{\partial x}\left(2xy^3\right) - \frac{\partial}{\partial y}\left(y^4\right)\,dA$$

$$= \iint_D 2y^3 - 4y^3\,dA \quad = \iint_D -2y^3\,dA$$

$$1 = \frac{x^2}{2} + y^2 = \underbrace{\left(\frac{x}{\sqrt{2}}\right)^2}_{u} + \underbrace{(y)^2}_{v} = u^2 + v^2$$



unit circle

$$\longrightarrow \quad x = \sqrt{2}\,u \quad \text{and} \quad y = v$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{vmatrix} = \sqrt{2}$$

$$= \iint_{u^2+v^2 \le 1} -2v^3 \cdot \left|\sqrt{2}\right|\,du\,dv$$

$$= \int_0^{2\pi} \int_0^1 -2\left(r\sin\theta\right)^3 \cdot \sqrt{2} \cdot r\,dr\,d\theta$$

$$= -2\sqrt{2}\int_0^{2\pi}\int_0^1 r^4 \sin^3\theta \, dr\, d\theta$$

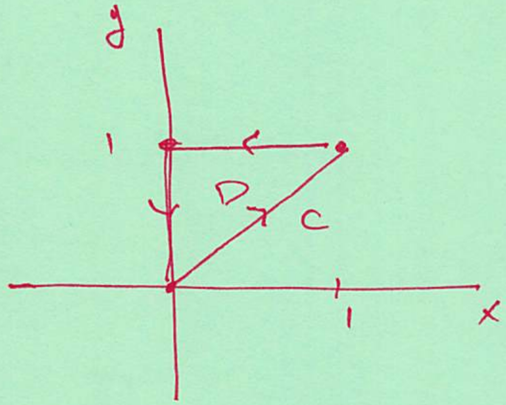
$$= -2\sqrt{2} \cdot \int_0^{2\pi} \sin^3\theta \, d\theta \cdot \int_0^1 r^4 \, dr$$

$$= -2\sqrt{2}\int_0^{2\pi} (1-\cos^2\theta)\sin\theta \, d\theta \cdot \left[\frac{r^5}{5}\right]_0^1$$

$$= \frac{-2\sqrt{2}}{5}\int_1^1 (1-u^2)(-du)$$

$$= 0.$$

\#14. $\vec{F} = \sqrt{x^2+1}\,\vec{i} + \tan^{-1}x\,\vec{j}$



$$\oint_C \vec{F}\cdot d\vec{r} = \iint_D \frac{\partial}{\partial x}\tan^{-1}x - \frac{\partial}{\partial y}\sqrt{x^2+1}\; dA$$

$$= \iint_D \frac{1}{1+x^2}\, dA$$

$$= \int_0^1 \int_x^1 \frac{1}{1+x^2}\, dy\, dx$$

$$= \int_0^1 \frac{1-x}{1+x^2}\, dx$$

$$= \tan^{-1}x \Big|_0^1 - \int_1^2 \frac{\frac{1}{2}du}{u}$$

$$= \pi/4 - 0 - \tfrac{1}{2}\ln 2 \quad = \pi/4 - \tfrac{1}{2}\ln 2.$$