16.6

#22. $x^2 + 2y^2 + 3z^2 = 1$

$$\Longrightarrow \quad x^2 + (\sqrt{2}\,y)^2 + (\sqrt{3}\,z)^2 = 1$$

Parametric equations:

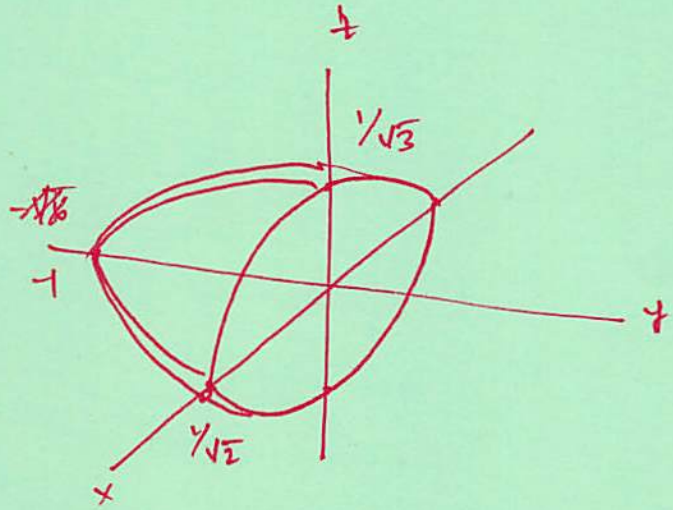
$$x = \cos\theta \sin\phi$$

$$y = \frac{1}{\sqrt{2}} \sin\theta \sin\phi$$

$$z = \frac{1}{\sqrt{3}} \cos\phi$$

Domain: $0 \le \phi \le \pi$

$\pi \le \theta \le 2\pi$



#48. $\vec{r}(u,v) = u\cos v\,\vec{i} + u\sin v\,\vec{j} + v\,\vec{k}$

$D: \quad 0 \le u \le 1, \quad 0 \le v \le \pi$

$\vec{r}_u = \cos v\,\vec{i} + \sin v\,\vec{j} + 0\vec{k}$

$\vec{r}_v = -u\sin v\,\vec{i} + u\cos v\,\vec{j} + \vec{k}$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos v & \sin v & 0 \\ -u\sin v & u\cos v & 1 \end{vmatrix}$$

$$= \vec{k}\left(u\cos^2 v + u\sin^2 v\right) - 0 + 1\cdot\left(\sin v\,\vec{i} - \cos v\,\vec{j}\right)$$

$$= \sin v\,\vec{i} - \cos v\,\vec{j} + u\vec{k}$$

$$\|\vec{r}_u \times \vec{r}_v\| = \sqrt{\sin^2 v + \cos^2 v + u^2} = \sqrt{1+u^2}$$

$$\text{Surface Area} = \iint_D \|\vec{r}_u \times \vec{r}_v\|\, dA = \int_0^{\pi}\int_0^1 \sqrt{1+u^2}\, du\, dv$$

trig sub: $u = \tan\theta$

$du = \sec^2\theta \, d\theta$

$\sqrt{1+u^2} = |\sec\theta|$

$1 = \tan\theta \iff \theta = \pi/4$

$0 = \tan\theta \iff \theta = 0$

$= \sec\theta$ (secant is positive on $0 \le \theta \le \pi/4$)

$$\Rightarrow \int_0^{\pi} \int_0^1 \sqrt{1+u^2} \, du \, dv$$

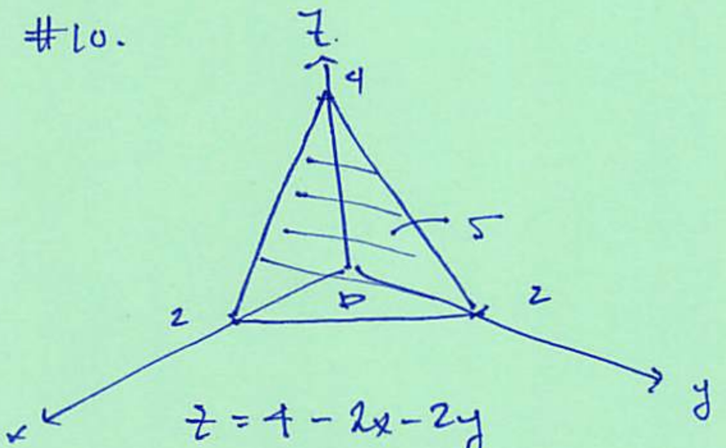
$$= \pi \cdot \int_0^{\pi/4} \sec\theta \cdot \sec^2\theta \, d\theta = \pi \int_0^{\pi/4} \sec^3\theta \, d\theta$$

$$= \pi \left[\frac{1}{2} \left(\sec\theta \tan\theta + \ln|\sec\theta + \tan\theta| \right) \right]_0^{\pi/4}$$

$$= \frac{\pi}{2} \left[\sqrt{2} + \ln(\sqrt{2}+1) - (0 + \ln(1)) \right]$$

$$= \frac{\pi}{2} \left(\sqrt{2} + \ln(\sqrt{2}+1) \right)$$

#10.



$z = 4 - 2x - 2y$

$D: \quad 0 \le x \le 2, \quad 0 \le y \le 2 - x$

$\vec{r}(x,y) = x\vec{i} + y\vec{j} + (4-2x-2y)\vec{k}$, D as above

parametrize S

$\vec{r}_x = \vec{i} + (-2)\vec{k}$

$\vec{r}_y = \vec{j} + (-2)\vec{k}$

$$\vec{r}_x \times \vec{r}_y = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -2 \\ 0 & 1 & -2 \end{vmatrix}$$

$= \vec{i}(2) - 1\left((-2)\vec{j} - \vec{k}\right)$

$= 2\vec{i} + 2\vec{j} + \vec{k}$

$$\|\vec{r}_x \times \vec{r}_y\| = \sqrt{4+4+1} = 3$$

$$f(x,y,z) = xz$$

$$\Rightarrow f(\vec{r}(x,y)) = f(x,y,4-2x-2y)$$

$$= x(4-2x-2y)$$

$$\iint_S xz\,dS = \iint_D x(4-2x-2y)\cdot 3\,dA$$

$$= 3\int_0^2\int_0^{2-x} 4x - 2x^2 - 2yx\,dy\,dx$$

$$= 3\int_0^2 (4x-2x^2)y - y^2x \Big|_0^{2-x} dx$$

$$= 3\int_0^2 2x(2-x)^2 - (2-x)^2 x\,dx$$

$$= 3\int_0^2 x(2-x)^2\,dx$$

$$= 3 \int_0^2 2x - 4x^2 + x^3 \, dx$$

$$= 3 \left[x^2 - \frac{4}{3} x^3 + \frac{1}{4} x^4 \right]_0^2$$

$$= 3 \left[4 - \frac{4}{3} \cdot 4 \cdot 2 + \frac{1}{\cancel{4}} \cdot \cancel{4} \cdot 4 \right]$$

$$= 12 \left[1 - \frac{8}{3} + 1 \right]$$

$$= 12 \left[\frac{6 - 8}{3} \right]$$

$$= 4 \cdot (-2)$$

$$= -8$$