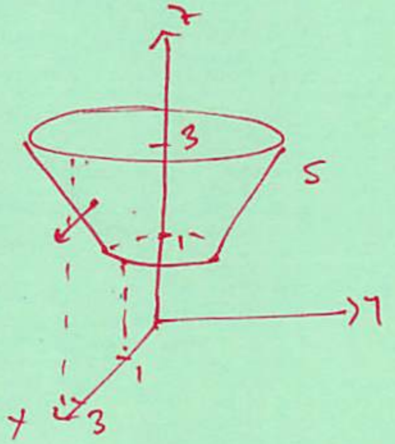
16.7

#24. $\vec{F} = -x\vec{i} - y\vec{j} + z^3\vec{k}$



Parametrize S using cylindrical coordinates

$\vec{r}(r,\theta) = r\cos\theta\,\vec{i} + r\sin\theta\,\vec{j} + r\,\vec{k}$

D: $1 \le r \le 3$

$0 \le \theta \le 2\pi$

$\vec{r}_r = \cos\theta\,\vec{i} + \sin\theta\,\vec{j} + \vec{k}$

$\vec{r}_\theta = -r\sin\theta\,\vec{i} + r\cos\theta\,\vec{j} + 0\vec{k}$

$$\vec{r}_r \times \vec{r}_\theta = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos\theta & \sin\theta & 1 \\ -r\sin\theta & r\cos\theta & 0 \end{vmatrix}$$

$$= \vec{k}\left(r\right) - 1\cdot\left(r\cos\theta\,\vec{i} + r\sin\theta\,\vec{j}\right)$$

$$= -r\cos\theta\,\vec{i} - r\sin\theta\,\vec{j} + r\vec{k}$$

$\vec{r}_r \times \vec{r}_\theta$ points "upward" we need to reverse the orientation.

typo

$$\vec{F}(\vec{r}(\vec{r},\theta)) = \vec{F}(r\cos\theta, r\sin\theta, r)$$

$$= -r\cos\theta\,\vec{i} - r\sin\theta\,\vec{j} + r^3\,\vec{k}$$

$$\vec{F}(\vec{r}(r,\theta)) \cdot (-\vec{r}_r \times \vec{r}_\theta)$$

$$= -r^2\cos^2\theta - r^2\sin^2\theta - r^4$$

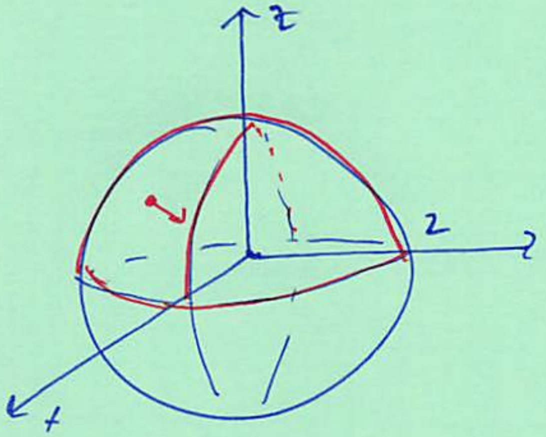
$$= -r^2 - r^4$$

$$\text{flux} = \iint_S \vec{F}\cdot d\vec{S} = \int_0^{2\pi}\int_1^3 -r^2 - r^4 \, dr\, d\theta$$

$$= -2\pi\left[\frac{r^3}{3} + \frac{r^5}{5}\right]_1^3$$

$$= -2\pi\left[\frac{3^3}{3} + \frac{3^5}{5} - \left(\frac{1}{3} + \frac{1}{5}\right)\right]$$

#26. $\vec{F} = y\vec{i} - x\vec{j} + 2z\vec{k}$



$\vec{r}(\phi, \theta) = \cos\theta \sin\phi\,\vec{i} + \sin\theta \sin\phi\,\vec{j} + \cos\phi\,\vec{k}$

$\vec{r}_\phi \times \vec{r}_\theta = \sin^2\phi \cos\theta\,\vec{i} + \sin^2\phi \sin\theta\,\vec{j} + \sin\phi \cos\theta\,\vec{k}$

see notes. we did this in class.

$\vec{F}(\vec{r}(\phi, \theta)) = \sin\phi \sin\theta\,\vec{i} - \sin\phi \cos\theta\,\vec{j} + 2\cos\phi\,\vec{k}$

but $\vec{r}_\phi \times \vec{r}_\theta$ points outward!

reverse it by multiplying by -1.

so

$$\vec{F}(\vec{r}(\phi, \theta)) \left(-\vec{r}_\phi \times \vec{r}_\theta\right)$$

$$= -\left(\sin^3\phi \sin\theta \cos\theta - \sin^3\phi \sin\theta \cos\theta + 2\sin\phi \cos\phi \cos\theta\right)$$

$$= -2\sin\phi \cos\phi \cos\theta$$

$\longrightarrow$

$$\text{flux} = \iint_S \vec{F} \cdot d\vec{S}$$

$$= \int_0^{2\pi} \int_0^{\pi/2} -2\sin\phi \cos\phi \cos\theta \, d\phi \, d\theta$$

full period of cos.

$$= 0.$$