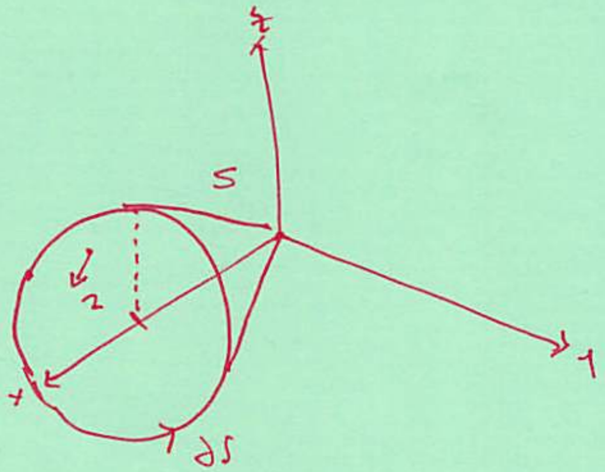
16.8

#4. $\vec{F} = \tan^{-1}(x^2yz^2)\,\vec{i} + x^2y\,\vec{j} + x^2z^2\,\vec{k}$



Param ∂S:

$\vec{r}(t) = 2\vec{i} + 2\cos t\,\vec{j} + 2\sin t\,\vec{k}$

$\vec{r}\,'(t) = -2\sin t\,\vec{j} + 2\cos t\,\vec{k}$

$$\vec{F}(\vec{r}(t)) = \tan^{-1}\left(4\cdot 2\cos t\cdot 4\sin^2 t\right)\vec{i} + 4\cdot 2\cos t\,\vec{j} + 4\cdot 4\sin^2 t\,\vec{k}$$

$$\vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t) = -16\sin t\cos t + 32\sin^2 t\cos t$$

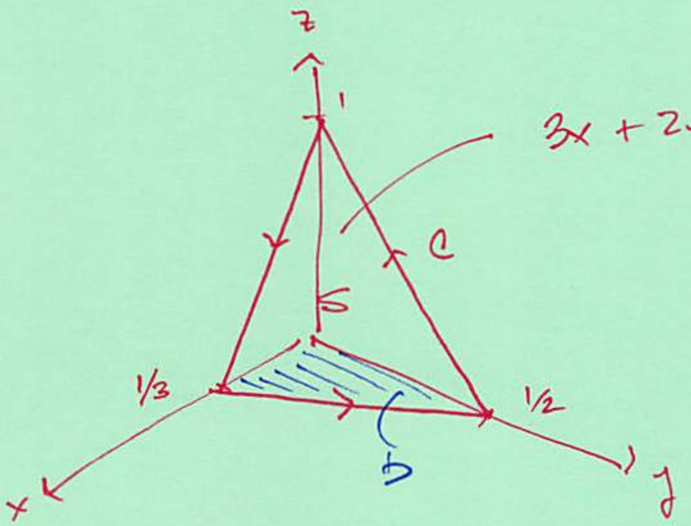
$$\iint_S \operatorname{curl}\vec{F}\cdot d\vec{S} = \int_{\partial S}\vec{F}\cdot d\vec{r}$$

$$= \int_0^{2\pi} -16\sin t\cos t + 32\sin^2 t\cos t\,dt$$

$u = \sin t \implies u(0) = 0 = u(2\pi)$

$= 0.$

#8. $\vec{F} = \vec{i} + (x+yz)\vec{j} + (xy - \sqrt{z})\vec{k}$



$3x + 2y + z = 1$

$z = 1 - 3x - 2y$

$\vec{r}(x,y) = x\vec{i} + y\vec{j} + (1-3x-2y)\vec{k}$

$D: \quad 0 \le x \le 1/3$

$\quad\;\; 0 \le y \le \dfrac{1-3x}{2}$

$\vec{r}_x = \vec{i} + (-3)\vec{k}, \quad \vec{r}_y = \vec{j} + (-2)\vec{k}$

$$\vec{r}_x \times \vec{r}_y = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -3 \\ 0 & 1 & -2 \end{vmatrix}$$

$$= 3\vec{i} + 2\vec{j} + \vec{k}$$

$$\operatorname{curl} \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 1 & x+yz & xy - \sqrt{z} \end{vmatrix}$$

$$= (x - y)\vec{i} - (y - 0)\vec{j} + (1 - 0)\vec{k}$$

$$= (x-y)\vec{i} - y\vec{j} + \vec{k}$$

$$\text{curl}\,\vec{F}(\vec{r}(x,y)) = (x-y)\vec{i} - y\vec{j} + \vec{k}$$

$$\text{curl}\,\vec{F}(\vec{r}(x,y)) \cdot \vec{r}_x \times \vec{r}_y$$

$$= 3(x-y) + 2(-y) + 1(1)$$

$$= 3x - 5y + 1$$

$$\int_C \vec{F} \cdot d\vec{r} = \iint_S \text{curl}\,\vec{F} \cdot d\vec{S}$$

$$= \iint_D 3x - 5y + 1 \, dA$$

$$= \int_0^{1/3} \int_0^{-3/2 x + 1/2} 3x - 5y + 1 \, dy\,dx$$

$$= \int_0^{1/3} (3x+1)y - \frac{5}{2}y^2 \Big|_0^{-3/2 x + 1/2} dx$$

$$= \int_0^{1/3} (3x+1)(-3x+1)\cdot\frac{1}{2} - \frac{5}{2}\left(1-3x\right)\cdot\frac{1}{2} \; dx$$

$$= \frac{1}{4}\int_0^{1/3} 2\cdot\left(1-9x^2\right) - 5\left(1-3x\right) dx$$

$$= \frac{1}{4}\int_0^{1/3} -18x^2 + 15x - 3 \; dx$$

$$= \frac{1}{4}\left[-6x^3 + \frac{15}{2}x^2 - 3x\right]_0^{1/3}$$

$$= \frac{1}{4}\left[-\frac{2}{9} + \frac{5}{6} - 1\right]$$

$$= \frac{1}{4}\left[\frac{-4+15-18}{18}\right] = \frac{1}{4}\left[-\frac{7}{18}\right]$$

$$= -\frac{7}{72}$$