Math 307 Exam 1, Fall 2022

Name:

Question	Points	Score
1	10	
2	9	
3	8	
4	4	
5	9	
6	9	
7	5	
Total:	54	

- You have 75 minutes to complete this exam.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work, unless otherwise indicated. You will get almost no credit for solutions that are not fully justified.
- You may not use notes or calculators on this exam.
- The back side of each page can be used as scratch work or for longer solutions. Please indicate clearly if your solution extends to the back side of a page.

1. (10 points) True/False questions. No justification necessary.

(a) True False If A is a 3×5 matrix and B is a 5×2, then AB is a 2×3 matrix.

(b) True False Every invertible matrix has nonzero determinant.

(c) True False For every n we have $\det(2I_n) = 2^n$.

(d) True False Any 2×2 matrix row reduces to one of the following four matrices:

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \text{ or } \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

(e) True False For any square matrices A and B,

$$(-A)B = A(-B) = -(AB).$$

(f) True False The matrix $A + A^T$ is symmetric for any square matrix A.

(g) True False A homogeneous system of equations $A\vec{x} = \vec{0}$ can be inconsistent.

(h) True False There is a vector space V such that $1 \cdot v = \vec{0}$ for every vector v in V.

(i) True False Multiplying a 3×3 matrix A on the left by

$$P_{1,2} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

has the effect of exchanging rows 2 and 3 in the matrix A.

(j) True False If the vectors $v_1, \ldots, v_n$ are a basis for V, then the vectors $v_1, \ldots, v_n$ are linearly dependent and span V.

2. (9 points) Multiple choice. No justification is necessary.

(a) Which of the following matrices are in RREF?

Matrix	Is in RREF	Is NOT in RREF
$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$		
$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$		
$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$		
$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$		
$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$		

(b) If

$$W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid P(a,b,c,d) \right\}$$

which of the following conditions $P(a,b,c,d)$ make W a subspace of $M_{2\times 2}(\mathbb{R})$?

$P(a,b,c,d)$	Is a subspace	Is NOT a subspace
The zero matrix: $a = b = c = d = 0$		
Upper-triangular matrices: $c = 0$		
The matrices with determinant 1: $ad - bc = 1$		
The scalar matrices: $a = d$, and $b = c = 0$		

3. (8 points) Let

$$A = \begin{bmatrix} 0 & 3 & -1 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}.$$

Compute A^{-1}, A^T, and A^2.

4. (4 points) Solve the linear system of equations

$$\begin{aligned} x_1 - 3x_2 + 12x_3 - x_4 + 7x_5 &= 3 \\ x_2 + 2x_3 - x_5 &= 2 \end{aligned}$$

5. (9 points) Let

$$v_1 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \text{ and } v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

(a) Are the vectors v_1 and v_2 linearly independent or linearly dependent? Justify.

(b) Is the vector $\begin{bmatrix} 0 \\ 0 \\ -8 \end{bmatrix}$ in the span of the vectors v_1 and v_2? Justify.

(c) Based on your findings in parts (a) and (b) do v_1 and v_2 form a basis for $\mathbb{R}^3$?

6. (9 points) Let

$$A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 10 & 5 & 3 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

(a) Find $\det(A)$, $\det(B)$, and $\det(C)$.

(b) Find $\det((A^T)^{-1})$.

(c) Find $\det((-A)B^2)$.

7. (5 points) Let A be a 3×3 matrix with $\det(A) = 10$.

(a) Consider the linear system $A\vec{x} = \vec{b}$. Suppose

$$\det(A_1) = 7, \quad \det(A_2) = 3, \quad \text{and} \quad \det(A_3) = 1,$$

where A_i is the matrix obtained from A by replacing the ith column vector of A with the vector $\vec{b}$. What is the solution to the linear system?

(b) If A has cofactor matrix

$$C = \begin{bmatrix} 2 & -2 & 2 \\ 2 & 3 & -3 \\ 0 & 10 & 0 \end{bmatrix},$$

what is the inverse of A?