Math 307 Exam 1, Spring 2023

Name:

Question	Points	Score
1	10	
2	0	
3	0	
4	0	
5	0	
6	0	
7	0	
8	0	
9	0	
Total:	10	

- You have 75 minutes to complete this exam.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work, unless otherwise indicated. You will get almost no credit for solutions that are not fully justified.
- No electronic devices are authorized with the exception of a scientific calculator.
- The back side of each page can be used as scratch work or for longer solutions. Please indicate clearly if your solution extends to the back side of a page.

1. (10 points) True/False questions. No justification necessary.

(a) True False There is a 5×6 matrix that row-equivalent to a matrix with 6 leading 1's.

(b) True False A system with 3 equations in 2 variables is always inconsistent.

(c) True False If A is a 3×2 matrix and B is a 3×2 matrix, then AB is a 3×2 matrix.

(d) True False If A is a square matrix, then $(2A)^4 = 16A^4$.

(e) True False If A is an invertible matrix, then the homogeneous system $A\vec{x} = \vec{0}$ only has the trivial solution.

(f) True False The matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

is invertible.

(g) True False The transpose of a 7×18 matrix is a matrix of size 18×7.

(h) True False For all $n \times n$ invertible matrices A and B, $(AB)^{-1} = B^{-1}A^{-1}$.

(i) True False If every entry of a 17×17 matrix is a 2, then the determinant of this matrix is 2^{17}.

(j) True False $\det(AA^T) = \det(A^2)$.

(k) True False The determinant of a triangular matrix is the product of the diagonal entries.

(l) True False For all square matrices A and B, $\det(A + B) = \det(A) + \det(B)$.

(m) True False There is a vector space V where $3 \cdot v = 2 \cdot v$ for all $v \in V$.

(n) True False The vector

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

is in the span of the vectors

$$\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 7 \\ 0 \\ 0 \end{bmatrix},$$

(o) True False Every vector space V has at least two subspaces, the zero vector space $\{\vec{0}\}$ and the space V itself.

(p) True False Every invertible matrix is a product of elementary matrices.

(q) True False To add 2 times row 3 onto row 1 of the 3×3 matrix A we multiply on the right by the matrix

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

(r) True False Every matrix in row-reduced echelon form has at least one 1 in one of its columns.

(s) True False Linear algebra is awesome!

2. The coefficiant matrix of each of the following augmented matrices is in row-reduced echelon form. In the space provided write the solution set (if it exists) to the corresponding system of linear equations

Matrix	Solution Set
$\left[\begin{array}{ccccc\|c} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$	
$\left[\begin{array}{ccccc\|c} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$	
$\left[\begin{array}{ccccc\|c} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{array}\right]$	

3. Prove or disprove that the following sets are subspaces of $M_3(\mathbb{R})$.
   (a) The set of all upper triangular 3×3 matrices.
   (b) The set of 3×3 matrices that have a 1 in the 1,1 entry.
   (c) The set of 3×3 invertible matrices.

4. Show that the matrix

$$A = \begin{bmatrix} 5 & 2 & 1 \\ 4 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

is invertible three different ways.

5. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

Compute the following or state that the expression is not defined.

(a) AC

(b) BC

(c) $A + C$

(d) $A + C^T$.

(e) $(BA)^T + 2C$

6. Let

$$A = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 2 & 4 & 11 \\ 0 & 0 & 3 & 17 \\ 0 & 0 & 0 & 1 \end{bmatrix} B = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 3 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 4 \end{bmatrix} C = \begin{bmatrix} 2 & 3 & 0 & 2 \\ 4 & 3 & 2 & 1 \\ 6 & 0 & 0 & 3 \\ 7 & 0 & 0 & 4 \end{bmatrix}$$

Compute the following determinants:

(a) $\det A$, $\det B$, and $\det C$.

(b) $\det(ABABAB)$, $\det(A^T B^{-1} C^2)$

7. Determine if the vector $v(x) = x^2 + 3x - 1$ is in the span of

$$\begin{aligned} v_1(x) &= x^2 + x \\ v_2(x) &= x + 1 \\ v_3(x) &= x^2 + 1 \end{aligned}$$

8. Solve the system of linear equations

$$\begin{aligned} x_1 + 7x_3 + 2x_6 &= 0 \\ 3x_3 + x_6 &= 0 \end{aligned}$$

9. Let $f, g, h \in C[-1, 1]$ be the functions

$$\begin{aligned} f(x) &= |x| \\ g(x) &= x^2 - 1 \end{aligned}$$

Find $(2f + 3g)(-1)$.