#2.

$$\left[\begin{array}{ccc|c} 2 & 1 & -2 & 0 \\ 2 & -1 & -2 & 0 \\ 1 & 2 & -4 & 0 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$\therefore x = y = z = 0.$

#5

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 2 & 1 & -1 & 0 \\ 4 & 1 & 5 & 0 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & -7 & 0 \\ 0 & 0 & 0 & 0 \\ d & d & f & \end{array}\right]$$

Let z be free and

$x = -3z$

$y = 7z$

#6

$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & 4 \\ 1 & 9 & -1 & 2 \\ 1 & -1 & 2 & 7 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 7/5 & 0 \\ 0 & 1 & -3/5 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$\Longrightarrow \quad 0 = 1$

system has no solution.

#21. Homogeneous system has more variables than equations so it has infinitely many solutions by thm 1.1.