#12

$$A = \left[\begin{matrix} 0 & -2 & 1 \\ 2 & 4 & -1 \\ 2 & 1 & 2 \end{matrix}\right]\begin{matrix} \\ \\ R_3 + (-1)R_2 \end{matrix}$$

$$\longrightarrow \left[\begin{matrix} 0 & -2 & 1 \\ 2 & 4 & -1 \\ 0 & -3 & 3 \end{matrix}\right]\begin{matrix} R_1 \leftrightarrow R_2 \\ \\ \\ \end{matrix} = U_{32}(-1)A$$

$$\longrightarrow \left[\begin{matrix} 2 & 4 & -1 \\ 0 & -2 & 1 \\ 0 & -3 & 3 \end{matrix}\right]\begin{matrix} R_1 + (2)R_2 \\ \\ -\frac{2}{3}R_3 \end{matrix} = P_{12}\, U_{32}(-1)\, A$$

$$\longrightarrow \left[\begin{matrix} 2 & 0 & 1 \\ 0 & -2 & 1 \\ 0 & 2 & -2 \end{matrix}\right]\begin{matrix} \\ \\ R_3 + R_2 \end{matrix} = D_3\left(-\frac{2}{3}\right) U_{12}(2)\, P_{12}\, U_{32}(-1)\, A$$

$$\longrightarrow \left[\begin{matrix} 2 & 0 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{matrix}\right]\begin{matrix} R_1 + R_3 \\ R_2 + R_3 \\ \\ \end{matrix} = U_{32}(1)\, D_3\left(-\frac{2}{3}\right) U_{12}(2)\, P_{12}\, U_{32}(-1)\, A$$

$$\longrightarrow \begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{array}{l} R_1/2 \\ R_2/-2 \\ R_3/-1 \end{array}$$

$$= U_{23}(1)\,U_{13}(1)\,U_{32}(1)\,D_3\left(-\tfrac{2}{3}\right)U_{12}(2)\,P_{12} \cdot U_{32}(-1)\,A$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= D_1\left(\tfrac{1}{2}\right)D_2\left(-\tfrac{1}{2}\right)D_3(-1)\,U_{23}(1)\,U_{13}(1)\,U_{32}(1)\,D_3\left(-\tfrac{2}{3}\right)U_{12}(2)\,P_{12} \cdot U_{32}(-1)\,A$$

$$\Longrightarrow \quad I = BA$$

product of all elementary matrices above

$$\Longrightarrow \quad A = B^{-1} I = B^{-1}$$

apply inverse rule for product and the formulas for the inverse of each elementary matrix

$$A = U_{32}(1)\,U_{12}(-2)\,D_3\left(-\tfrac{3}{2}\right)U_{32}(-1)\,U_{13}(-1)\,U_{23}(-1)$$

$$\cdot\; D_3(-1)\,D_2(-2)\,D_1(2)$$

#11

$$(AB)^T = \left(\begin{bmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 3 & 5 \\ -4 & 1 \end{bmatrix} \right)^T$$

$$= \begin{bmatrix} 16 & 8 \\ -12 & -8 \end{bmatrix}^T$$

$$= \begin{bmatrix} 16 & -12 \\ 8 & -8 \end{bmatrix}$$

#12

$$B^T A^T = \begin{bmatrix} -2 & 1 \\ 3 & 5 \\ -4 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} -2 & 3 & -4 \\ 1 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & -12 \\ 8 & -8 \end{bmatrix}$$