1.3.

#20. x^2-3x and $x+7$ do not span P_2

Say

$$c_1(x^2-3x)+c_2(x+7)=ax^2+bx+c$$

$$\Longleftrightarrow \quad c_1x^2+(-3c_1+c_2)x+7c_2=ax^2+bx+c$$

$$\Longleftrightarrow \quad \begin{aligned} c_1 &= a \\ -3c_1+c_2 &= b \\ 7c_2 &= c \end{aligned}$$

From this system we can see the vector $v(x)=x$ is not in the span since the system

$$\begin{aligned} c_1 &= 0 \\ -3c_1+c_2 &= 1 \\ 7c_2 &= 0 \end{aligned}$$

Doesn't have a solution.

#26. Let

$$\gamma = \{ x^3+x,\ x^2-x,\ x+1,\ x^3+1 \}$$

and let $v(x) = x^3+x^2+x+1$.

$$c_1(x^3+x) + c_2(x^2-x) + c_3(x+1) + c_4(x^3+1) = v(x)$$

$$\iff \begin{aligned} c_1 + c_4 &= 1 \\ c_2 &= 1 \\ c_1 + (-c_2) + c_3 &= 1 \\ c_3 + c_4 &= 1 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & -1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right] \xrightarrow{RR} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\therefore [v]_\gamma = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

(b) If $[v]_\gamma = \begin{bmatrix} -2 \\ 2 \\ 0 \\ 1 \end{bmatrix}$, then

$$v(x) = -2\left(x^3+x\right) + 2\left(x^2-x\right) + 0\left(x+1\right) + 1\left(x^3+1\right)$$

$$= -x^3 + 2x^2 - 4x + 1$$

2.4

#2.

(a) Since

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 1 & -1 & -1 & 0 \\ 4 & 0 & 8 & 0 \end{array}\right] \xrightarrow{RR} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$\Rightarrow$ the columns are L.I.

$\Rightarrow$ the columns are a basis since $\dim(\mathbb{R}^3) = 3$. (thm 2.12)

(b) Since

$$\left[\begin{array}{ccc|c} 3 & -1 & 3 & 0 \\ 2 & -1 & 1 & 0 \\ 1 & 0 & 2 & 0 \end{array}\right] \xrightarrow{RR} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\Longrightarrow$ the system has nontrivial solutions

$\Rightarrow$ the columns are L.D.

$\therefore$ not a basis.

(c & d) The dimension of $\mathbb{R}^3$ is 3.

Since neither list of vectors has 3 vectors

they cannot possibly form a basis (thm 2.9)