5.1

#9. $T : F(\mathbb{R}) \to F(\mathbb{R})$

$$T(f(x)) = f(x) - 2$$

This map is not linear: let

$f(x) = e^x$ and $g(x) = 2$, then

$$T(f(x) + g(x)) = T(e^x + 2)$$

$$= (e^x + 2) - 2$$

$$= e^x$$

On the other hand,

$$T(f(x)) + T(g(x)) = \cancel{e}$$

$$= T(e^x) + T(2)$$

$$= (e^x - 2) + (2 - 2)$$

$$= e^x - 2$$

$$\therefore T(f(x) + g(x)) \neq T(f(x)) + T(g(x)).$$

#10. $T : F(\mathbb{R}) \to F(\mathbb{R})$

$$T(f(x)) = f(x-2)$$

Linearity: Let $f, g \in F(\mathbb{R})$, then

$$T(f(x)+g(x)) = T((f+g)(x))$$

$$= (f+g)(x-2)$$

$$= f(x-2) + g(x-2)$$

$$= T(f(x)) + T(g(x))$$

Homogeneity: Let $f \in F(\mathbb{R})$ and $c \in \mathbb{R}$, then

$$T(c \cdot f(x)) = T((cf)(x))$$

$$= (cf)(x-2)$$

$$= c \cdot f(x-2)$$

$$= c \cdot T(f(x))$$

#12. $T : M_n(\mathbb{R}) \to \mathbb{R}$

$$T(A) = \det(A)$$

This map is not homogeneous if $n > 1$.

Let $A = I$ and $c = 2$, then

$$T(2I) = \det(2I) = 2^n \cdot \det(I)$$

$$= 2^n \cdot 1$$

$$= 2^n$$

On the other hand,

$$2 \cdot T(I) = 2 \cdot \det(I) = 2 \cdot 1 = 2.$$

$$\therefore \quad T(2I) \neq 2 \cdot T(I).$$

However, if $n = 1$, then $M_1(\mathbb{R})$ is basically just $\mathbb{R}$.

$\longrightarrow$

So

$$T : M_1(\mathbb{R}) \to \mathbb{R}$$

$$T([a]) = \det[a] = a$$

Hence, T is just the identity operator which is a linear transformation.