

Solutions 5.3

$$\#6 \quad D: P_2 \rightarrow P_2; \quad D(ax^2 + bx + c) = 2ax + b$$

$$\alpha = \{x^2, x, 1\}$$

$$\beta = \{x^2+1, x+1, 2x^2+1\}$$

$$v = x^2 + 2x - 2$$

$$\begin{aligned} (a) \quad [D]_{\alpha}^{\alpha} &= \begin{bmatrix} [Dx^2]_{\alpha} & [Dx]_{\alpha} & [D1]_{\alpha} \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} (b) \quad P &= [I]_{\beta}^{\alpha} = \begin{bmatrix} [x^2+1]_{\alpha} & [x+1]_{\alpha} & [2x^2+1]_{\alpha} \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \end{aligned}$$

$$(c) \quad P^{-1} = \begin{bmatrix} -1 & -2 & 2 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

$$(a) \quad [D]_{\beta}^{\beta} = P^{-1} [D]_{\alpha}^{\alpha} P$$

$$= \begin{bmatrix} -1 & -2 & 2 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 2 & -8 \\ 2 & 0 & 4 \\ 2 & -1 & 4 \end{bmatrix}$$

$$(r) \quad [y]_{\beta} = P^{-1} [y]_{\alpha} = \begin{bmatrix} -1 & -2 & 2 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} -9 \\ 2 \\ 5 \end{bmatrix}$$

$$(s) \quad [Dy]_{\beta} = [D]_{\beta}^{\beta} [y]_{\beta} = \begin{bmatrix} -4 & 2 & -8 \\ 2 & 0 & 4 \\ 2 & -1 & 4 \end{bmatrix} \begin{bmatrix} -9 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$(t) \quad Dy = 0(x^2+1) + 2(x+1) + 0(2x^2+1) \\ = 2x+2$$

#10 $\alpha = \{v_1, v_2, v_3, v_4\}$ basis for V

$\beta = \{w_1, w_2\}$ basis for W

$T: V \rightarrow W$

$$Tv_1 = 2w_1 - 3w_2, \quad Tv_2 = -w_1 + 3w_2$$

$$Tv_3 = w_1 + 2w_2, \quad Tv_4 = 3w_2$$

$$\begin{aligned} (a) \quad [T]_{\alpha}^{\beta} &= \begin{bmatrix} [Tv_1]_{\beta} & [Tv_2]_{\beta} & [Tv_3]_{\beta} & [Tv_4]_{\beta} \end{bmatrix} \\ &= \begin{bmatrix} [2w_1 - 3w_2]_{\beta} & [-w_1 + 3w_2]_{\beta} & [w_1 + 2w_2]_{\beta} & [3w_2]_{\beta} \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 2 & -1 & 1 & 0 \\ -3 & 3 & 2 & 3 \end{bmatrix}$$

$$(b) \quad [Tv]_{\beta} = [T]_{\alpha}^{\beta} [v]_{\alpha} = \begin{bmatrix} 2 & -1 & 1 & 0 \\ -3 & 3 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$

$$(c) \quad Tv = 7w_1 + 4w_2$$