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$$\#2. \quad A = \begin{bmatrix} 4 & -4 \\ 1 & 0 \end{bmatrix}$$

$$p(\lambda) = \det(\lambda I - A) = \begin{vmatrix} \lambda - 4 & 4 \\ -1 & \lambda \end{vmatrix}$$

$$= \lambda^2 - 4\lambda + 4$$

$$= (\lambda - 2)^2, \text{ eigvals } 2 \text{ ~~times~~ (mult 2)}$$

$$\lambda = 2: \quad \left[2I - A \mid \vec{0} \right] = \left[\begin{array}{cc|c} -2 & 4 & 0 \\ -1 & 2 & 0 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

let y be free, then $x = 2y$

$$\begin{bmatrix} x \\ y \end{bmatrix} \in E_2 \iff \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2y \\ y \end{bmatrix} = y \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

\uparrow
basis for E_2

$$\#5. \quad A = \begin{bmatrix} 0 & 3 \\ 4 & 0 \end{bmatrix}$$

$$p(\lambda) = \begin{vmatrix} \lambda & -3 \\ -4 & \lambda \end{vmatrix} = \lambda^2 - 12 = 0$$

$$\Rightarrow \lambda = \pm \sqrt{12} = \pm 2\sqrt{3}$$

↖ eigenvalues

$$\lambda = 2\sqrt{3} :$$

$$\left[2\sqrt{3} - A \mid \vec{0} \right] = \left[\begin{array}{cc|c} 2\sqrt{3} & -3 & 0 \\ -4 & 2\sqrt{3} & 0 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{cc|c} 1 & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 \end{array} \right]$$

let y be free, then $x = \frac{\sqrt{3}}{2}y$.

$$\begin{bmatrix} x \\ y \end{bmatrix} \in E_{2\sqrt{3}} \iff \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2}y \\ y \end{bmatrix} = y \begin{bmatrix} \sqrt{3}/2 \\ 1 \end{bmatrix}$$

↗ basis for
 ~~$E_{2\sqrt{3}}$~~ $E_{2\sqrt{3}}$

$$\lambda = -2\sqrt{3}!$$

$$\left[-2\sqrt{3} I - A \mid \vec{0} \right] = \left[\begin{array}{cc|c} -2\sqrt{3} & -3 & 0 \\ -4 & -2\sqrt{3} & 0 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{cc|c} 1 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

$d \quad \neq$

Let y be free so $x = -\sqrt{3}/2 y$

$$\begin{bmatrix} x \\ y \end{bmatrix} \in \left\{ \begin{array}{c} \sqrt{3}/2 \\ -2\sqrt{3} \end{array} \right\} \Leftrightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\sqrt{3}/2 y \\ y \end{bmatrix}$$

$$= y \begin{bmatrix} -\sqrt{3}/2 \\ 1 \end{bmatrix}$$

↗
basis for $E_{-2\sqrt{3}}$