5.4 / 5.5

#8. $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}$

$$p(\lambda) = \begin{vmatrix} \lambda-1 & -1 & 0 \\ 0 & \lambda-1 & 1 \\ 0 & 0 & \lambda-2 \end{vmatrix} = (\lambda-1)^2(\lambda-2) = 0$$

eigvals $\lambda = 1, 2$

$\lambda = 1$: $\left[\begin{array}{ccc|c} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

f d d

x free, $y = z = 0$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \in E_1 \iff \begin{bmatrix} x \\ y \\ z \end{bmatrix} = x e_1$ basis for E_1

$\lambda = 2$: $\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

d d f

z free, $x = y = -z$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \in E_2 \iff \begin{bmatrix} x \\ y \\ z \end{bmatrix} = z \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

basis fr E_2

$$\dim(E_1) + \dim(E_2) = 2 \neq 3$$

$\therefore$ A is not diaganalizable.

#10. $A = \begin{bmatrix} -6 & 0 & -8 \\ -4 & 2 & -4 \\ 4 & 0 & 6 \end{bmatrix}$

$$p(\lambda) = \begin{vmatrix} \lambda+6 & 0 & 8 \\ 4 & \lambda-2 & 4 \\ -4 & 0 & \lambda-6 \end{vmatrix}$$

$$= (\lambda-2)\left[(\lambda+6)(\lambda-6) + 32\right]$$

$$= (\lambda-2)\left[\lambda^2 - 36 + 32\right]$$

$$\lambda^2 - 4$$

$$= (\lambda-2)^2(\lambda+2) = 0$$

Eigvals $\lambda = 2, -2$

$$\lambda = -2: \quad \left[\begin{array}{ccc|c} 4 & 0 & 8 & 0 \\ 4 & -6 & 4 & 0 \\ -4 & 0 & -8 & 0 \end{array}\right] \longrightarrow \begin{array}{c} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 2/3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \\ \begin{array}{cccc} d & d & f & \end{array} \end{array}$$

z free, $x = -2z$, $y = -\frac{2}{3}z$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \in E_{-2} \iff \begin{bmatrix} x \\ y \\ z \end{bmatrix} = z \begin{bmatrix} -2 \\ -2/3 \\ 1 \end{bmatrix}$$

basis for E_{-2}

$$\lambda = 2: \quad \left[\begin{array}{ccc|c} 8 & 0 & 8 & 0 \\ 4 & 0 & 4 & 0 \\ -4 & 0 & -4 & 0 \end{array}\right] \longrightarrow \begin{array}{c} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \\ \begin{array}{cccc} d & f & f & \end{array} \end{array}$$

y, z free and $x = -z$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \in E_2 \iff \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -z \\ y \\ z \end{bmatrix} = y \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

basis for E_2

$\dim(E_{-2}) + \dim(E_2) = 3$ so A is diagonalizable.

$$D = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \quad P = \begin{bmatrix} -2 & 0 & -1 \\ -2/3 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$