Math 242 Final Fall 2018

By Kenny

Name: Solutions

(Please circle your section)

Section 1, MWF 12:30-1:20, Jeffrey Lyons

Section 2, MWF 12:30-1:20, Jeffrey Lyons

Section 3, MWF 1:30-2:20, Jeffrey Lyons

Section 4, MWF 1:30-2:20, Jeffrey Lyons

Section 5, TR 9:00-10:15, Anne Carter

Section 6, TR 9:00-10:15, Anne Carter

Section 7, TR 12:00-1:15, Elliot Ossanna

Section 8, TR 12:00-1:15, Elliot Ossanna

Page	Points	Score
2	20	
3	20	
4	30	
5	30	
6	30	
7	25	
8	30	
9	30	
10	28	
11	17	
Total:	260	

- You may *not* use notes or calculators on the test.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work and make clear what your final solution is (e.g. by drawing a box around it), except for the "short-answer" problems at the end.
- If you run out of room on a problem, continue on the blank page (page 12) and indicate clearly that you are doing this.
- On problems with choices (like Problem 10) do not do more parts than we ask you to, or you might be penalized!
- The last two pages are a formula sheet. You are welcome to remove this from the exam.
- **Don't forget to put your name on the paper!**
- Good luck!

1. (10 points) Find $f'(x)$ where $f(x) = \frac{x^2 e^x}{\sqrt{1+x^2}}$.
(Hint: use logarithmic differentiation.)

$$\ln f(x) = \ln\left(\frac{x^2 e^x}{\sqrt{1+x^2}}\right) = \ln(x^2 e^x) - \ln\sqrt{1+x^2}$$

$$\ln f(x) = \ln x^2 + \ln e^x - \ln\sqrt{1+x^2}$$

$$\ln f(x) = 2\ln x + x - \frac{1}{2}\ln(1+x^2)$$

$$\Longrightarrow \frac{f'(x)}{f(x)} = \frac{2}{x} + 1 - \frac{1}{2}\frac{1}{1+x^2}(2x)$$

$$\Longrightarrow f'(x) = \frac{x^2 e^x}{\sqrt{1+x^2}}\left(\frac{2}{x} + 1 - \frac{1}{2}\frac{1}{1+x^2}(2x)\right)$$

2. (10 points) $\lim_{t\to 0} \frac{e^t - t - 1}{\sin^2(t)} = ?$ Type $\frac{0}{0}$

$$\overset{H}{=} \lim_{t\to 0} \frac{e^t - 1}{2\sin t\cos t} \quad \text{Type } \frac{0}{0}$$

$$\overset{H}{=} \lim_{t\to 0} \frac{e^t}{2(\cos^2 t - \sin^2 t)}$$

$$= \frac{e^0}{2(\cos^2 0 - \sin^2 0)}$$

$$= \frac{1}{2(1-0)} = \frac{1}{2}$$

3. (20 points) (a) Find $\displaystyle\int_3^\infty \frac{dx}{x(\ln x)^2}$ or show that it diverges.

$$= \lim_{t\to\infty} \int_3^t \frac{dx}{x(\ln x)^2} \qquad u = \ln x \qquad u(3) = \ln 3$$

$$du = \cancel{\ln t}\ \frac{1}{x}dx \qquad u(t) = \ln t$$

$$= \lim_{t\to\infty} \int_{\ln 3}^{\ln t} \frac{du}{u^2} = \lim_{t\to\infty} \int_{\ln 3}^{\ln t} u^{-2}\, du$$

$$= \lim_{t\to\infty} \left.\frac{u^{-1}}{-1}\right|_{\ln 3}^{\ln t}$$

$$= \lim_{t\to\infty} \left[\frac{1}{\ln 3} - \frac{1}{\ln t}\right]$$

$$= \frac{1}{\ln 3}$$

(b) What does this say about $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^2}$? (Fullest credit only if you verify all the conditions required for any tests you mention!)

Let $f(x) = \dfrac{1}{x(\ln x)^2}$ for $x > 0$, then f is

- positive: product of positive functions
- continuous: composed of continuous functions
- decreasing: denominator is unbounded.
- interpolates: $f(n) = \dfrac{1}{n(\ln n)^2}$

Then the integral test says $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^2}$ converges

as $\displaystyle\int_3^\infty \frac{1}{x(\ln x)^2}\,dx$ converges.

4. (30 points) Evaluate TWO of the following THREE integrals here. DO NOT DO ALL THREE! Cross out the one you do **not** want graded.

(a) $\displaystyle\int \tan^2(x)\sec^4(x)dx$ (b) $\displaystyle\int_0^{\sqrt{2}} \frac{x^3dx}{(4-x^2)^{3/2}}$ (c) $\displaystyle\int \frac{1}{e^x+e^{-x}}dx$

(a) EVEN power of sec

$u = \tan x$

$du = \sec^2 x\, dx$

$\sec^2 x = 1 + \tan^2 x = 1 + u^2$

$$= \int \tan^2 x \,(\sec^2 x)\,(\sec^2 x\, dx)$$

$$= \int u^2(1+u^2)\,du = \int u^2 + u^4\, du$$

$$= \frac{u^3}{3} + \frac{u^5}{5} + C = \boxed{\frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + C}$$

(b) trig sub

$x = 2\sin\theta$

$x^3 = 8\sin^3\theta$

$dx = 2\cos\theta\, d\theta$

$(4-x^2)^{3/2} = (4 - 4\sin^2\theta)^{3/2}$

$= 8(\cos^2\theta)^{3/2}$

$= 8\cos^3\theta$

$$\int_0^{\sqrt{2}} \frac{x^3}{(4-x^2)^{3/2}}dx = \int_0^{\pi/4} \frac{8\sin^3\theta}{8\cos^3\theta}\cdot 2\cos\theta\, d\theta$$

$$= 2\int_0^{\pi/4} \frac{(1-\cos^2\theta)}{\cos^2\theta}\sin\theta\, d\theta$$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$u(\pi/4) = \frac{\sqrt{2}}{2}$

$u(0) = 1$

$$= 2\int_1^{\sqrt{2}/2} \frac{1-u^2}{u^2}(-du)$$

$$= 2\int_{\sqrt{2}/2}^{1} u^{-2} - 1\, du = -2\left[u^{-1} - u\right]_{\sqrt{2}/2}^{1}$$

$$= -2\left[\sqrt{2} - \frac{1}{\sqrt{2}}\right]$$

(c) $\displaystyle\int \frac{1}{e^x + \frac{1}{e^x}}dx$

$$= \int \frac{e^x}{e^{2x}+1}dx$$

$$= \int \frac{e^x}{(e^x)^2+1}dx$$

$$\int \frac{du}{u^2+1}$$

$$= \tan^{-1} u + C$$

$$= \tan^{-1}(e^x) + C$$

$u = e^x$

$du = e^x dx$

5. (30 points) Evaluate TWO of the following THREE integrals. DO NOT DO ALL THREE! Cross out the one you do **not** want graded.

(a) $\int \frac{x+\sin^{-1}(x)}{\sqrt{1-x^2}}dx = ?$ (b) $\int \frac{1-x}{x^4+x^2}dx = ?$ (c) $\int e^t \cos(2t)dt = ?$

(a) $= \int \frac{x}{\sqrt{1-x^2}}dx + \int \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$ $u = 1-x^2$, $du = -2x\,dx$, $-\frac{1}{2}du = x\,dx$; $v = \sin^{-1}x$, $dv = \frac{1}{\sqrt{1-x^2}}dx$

$= \int \frac{-\frac{1}{2}du}{u^{1/2}} + \int v\,dv = -\frac{1}{2}\left[2u^{1/2}\right] + \frac{v^2}{2} + C$

$= -(1-x^2)^{1/2} + \frac{(\sin^{-1}x)^2}{2} + C$

(b) Partial Fractions

$\frac{1-x}{x^2(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}$

$1-x = Ax(x^2+1) + B(x^2+1) + (Cx+D)x^2$

$= Ax^3 + Ax + Bx^2 + B + Cx^3 + Dx^2$

$A+C=0$
$B+D=0$
$A=-1 \Rightarrow D=-1$
$B=1 \quad C=1$

$\int \frac{1-x}{x^2(x^2+1)}dx = \int -\frac{1}{x}dx + \int \frac{1}{x^2}dx + \int \frac{x-1}{x^2+1}dx$

$= -\ln|x| - x^{-1} + \frac{1}{2}\ln(x^2+1) - \tan^{-1}x + C$

(c)
$I = \int e^t \cos(2t)\,dt = e^t\cos 2t + \int (e^t)(2\sin 2t)\,dt$

$u = \cos 2t \quad dv = e^t dt$
$du = -2\sin 2t\,dt \quad v = e^t$

$= e^t\cos 2t + 2\left[e^t \sin 2t - \underbrace{\int (e^t)(2\cos 2t)\,dt}_{I}\right]$

$u = \sin 2t \quad dv = e^t dt$
$du = 2\cos 2t\,dt \quad v = e^t$

$\Rightarrow I = e^t\cos 2t + 2e^t\sin 2t - 2I$

$3I = e^t\cos 2t + 2e^t \sin 2t$

$I = \frac{e^t\cos 2t + 2e^t\sin 2t}{3}$

6. (10 points) Test for convergence: $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{(n+2)!}$

Ratio test!

$$\left| \frac{3^{n+1}}{(n+3)!} \cdot \frac{(n+2)!}{3^n} \right| = \left| \frac{3}{n+3} \right| = \frac{3}{n+3} \rightarrow 0 < 1$$

$\therefore$ series converges

7. (20 points) Compute the exact value (or show diverges):

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n}}{9^n} = \sum_{n=1}^{\infty} \left(\frac{4}{9}\right)^n = \frac{\text{first term}}{1 - \text{ratio}}$

$$= \frac{4/9}{1 - 4/9} = \frac{4}{5}$$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n^{1.5}} - \frac{1}{(n+1)^{1.5}} \right)$ (Hint:)

$$S_n = \left(\frac{1}{1^{3/2}} - \frac{1}{2^{3/2}} \right) + \left(\frac{1}{2^{3/2}} - \frac{1}{3^{3/2}} \right) + \cdots + \left(\frac{1}{n^{3/2}} - \frac{1}{(n+1)^{3/2}} \right)$$

$$= 1 - \frac{1}{(n+1)^{3/2}}$$

$$\rightarrow 1.$$

8. (10 points) Do ONE of the following TWO problems. Cross out the one you do **not** want graded.

(a) $\int \sin(2\theta)\cos(5\theta)d\theta = ?$ $\frac{1}{2}\int \sin(-3\theta) + \sin(7\theta)\, d\theta = \frac{\cos(3\theta)}{6} - \frac{\cos(7\theta)}{14} + C$

(b) Approximate $\int_1^2 \frac{1}{x}\, dx$ using Simpson's Rule and the Trapezoid Rule with $n = 4$. You need not simplify your answer.

$a = 1, \quad b = 2, \quad n = 4, \quad \Delta x = \frac{b-a}{n} = \frac{1}{4}$

x_i	1	5/4	6/4	7/4	2
$y_i = \frac{1}{x_i}$	1	4/5	4/6	4/7	1/2
Coeffs S_4	1	4	2	4	1
T_4	1	2	2	2	1

$$S_4 = \frac{1/4}{3}\left(1 + 4\cdot 4/5 + 2\cdot 4/6 + 4\cdot 4/7 + 1\cdot 1/2\right)$$

$$T_4 = \frac{1/4}{2}\left(1 + 2\cdot 4/5 + 2\cdot 4/6 + 2\cdot 4/7 + 1\cdot 1/2\right)$$

9. (15 points) Find the **radius** and **interval** of convergence for $\sum_{n=0}^{\infty} \frac{5n^2}{e^n} x^n$

Ratio test:

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{5(n+1)^2 x^{n+1}}{e^{n+1}} \cdot \frac{e^n}{5n^2 x^n}\right| = \left(\frac{n+1}{n}\right)^2 \cdot \frac{1}{e} \cdot |x| \to \frac{|x|}{e}$$

$$\frac{|x|}{e} < 1 \iff |x| < e \iff -e < x < e$$

If $x = \pm e$, both diverge by the divergence test.

$R = e$

I.O.C. $= (-e, e)$

10. (30 points) Solve TWO of the following THREE problems. DO NOT DO ALL THREE!. Cross out the one you do **not** want graded. You do not need to solve for y in terms of x or t in your answers.

   (a) ~~Solve the initial value problem: $y'' + 4y' + 4y = 0$; $y(0) = 1, y'(0) = 0$~~ not covered

   (b) Find the general solution to: $\csc(x^2+1)\frac{dy}{dx} = xy$ separable

   (c) Find the general solution to: $t\frac{dy}{dt} = t^2 + 3y \quad (t > 0)$ linear

(b) $\frac{1}{y}\,dy = \sin(x^2+1)\,x\,dx$ $\qquad u = x^2+1$, $du = 2x\,dx$

$$\int \frac{1}{y}\,dy = \int \sin(x^2+1)\,x\,dx$$

$$\ln|y| = \int \sin(u)\,\tfrac{1}{2}\,du = -\tfrac{1}{2}\cos(x^2+1) + C$$

$$|y| = e^{-\frac{1}{2}\cos(x^2+1)+C}$$

$$y = \pm e^{-\frac{1}{2}\cos(x^2+1)+C} \qquad (y = 0 \text{ is a solution})$$

$$\Rightarrow y = Ae^{-\frac{1}{2}\cos(x^2+1)}$$

(c) $y' - \frac{3}{t}y = t$

$$p(t) = -\frac{3}{t}$$

Integrating factor: $e^{\int p(t)\,dt} = e^{\int -\frac{3}{t}\,dt} = e^{-3\ln t} = t^{-3}$

$$\Rightarrow t^{-3}y' - t^{-3}\cdot\frac{3}{t}y = t^{-2}$$

$$(t^{-3}y)' = t^{-2}$$

$$t^{-3}y = \int t^{-2}\,dt = -t^{-1} + C$$

$$\Rightarrow y = t^3(-t^{-1} + C)$$

11. (30 points) Solve TWO of the following THREE problems. DO NOT DO ALL THREE! Cross out the one you do **not** want graded.

(a) Approximate the function $f(x) = \sqrt[3]{x}$ by a Taylor polynomial of degree 2 at the point $a = 8$. How accurate is this approximation when $7 \le x \le 9$? (You don't need to simplify your answer.)

(b) Find the power series for $f(x) = \dfrac{x}{2x^2+1}$, and determine the radius of convergence.

(c) Derive the formula for $\frac{d}{dx}\cos^{-1} x$. (Don't just quote the answer from the formula sheet, we need to see where the formula comes from.)

(a) $f(x) = x^{1/3} \Rightarrow f(8) = 2$

$f'(x) = \frac{1}{3}x^{-2/3} \Rightarrow f'(8) = \frac{1}{3}\,\frac{1}{4} = \frac{1}{12}$

$f''(x) = -\frac{2}{9}x^{-5/3} \Rightarrow f''(8) = -\frac{2}{9}\,\frac{1}{32} = -\frac{1}{144}$

$f'''(x) = \frac{10}{27}x^{-8/3}$

$f^{(4)}(x) = -\frac{80}{81}x^{-11/3}$

$$\boxed{T_2(x) = 2 + \tfrac{1}{12}(x-8) + \frac{-1/144}{2!}(x-8)^2}$$

$|f'''(x)| = \frac{10}{27}x^{-8/3} \le \frac{10}{27}\,7^{-8/3}$

f''' is positive and decreasing ($f^{(4)} < 0$) on $7 \le x \le 9$.

$7 \le x \le 9 \iff |x-8| \le 1$

$$\Rightarrow |R_2(x)| \le \frac{10}{27}\cdot 7^{-8/3}\,\frac{|x-8|^3}{3!} \le \frac{10}{27}\,7^{-8/3}\cdot\frac{1}{3!}$$

$$f(x) = x\cdot\frac{1}{(\sqrt{2}x)^2+1} = \frac{x}{\sqrt{2}}\,\frac{d}{dx}\tan^{-1}(\sqrt{2}x) = \frac{x}{\sqrt{2}}\,\frac{d}{dx}\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}$$

$$= \frac{x}{\sqrt{2}}\sum_{n=1}^{\infty}(-1)^n\frac{(2n+1)x^{2n}}{(2n+1)} = \sum_{n=1}^{\infty}(-1)^n\frac{x}{\sqrt{2}}\cdot x^{2n}$$

$$= \sum_{n=1}^{\infty}(-1)^n\frac{x^{2n+1}}{\sqrt{2}}$$

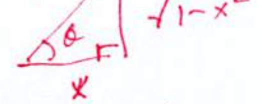
$R = 1.$

$$\frac{d}{dx}\cos^{-1}x = \frac{1}{\cos'(\cos^{-1}x)}$$

$$= \frac{1}{-\sin(\cos^{-1}(x))}$$

$$= -\frac{1}{\sqrt{1-x^2}}$$

$\theta = \cos^{-1}x \iff \cos\theta = x$



$\sin\theta = \sqrt{1-x^2}$

The problems on this page are short answer. You need not show work. They should all be *very quick*, if not you are doing it wrong!

12. (10 points) Circle all series that converge.

A. $\sum_{n=1}^{\infty} \frac{1}{n^2-n}$ B. $\sum_{n=1}^{\infty} \frac{n-1}{n+1}$ C. $\sum_{n=3}^{\infty} \frac{\ln(n)}{n^3}$ D. $\sum_{n=2}^{\infty} \frac{n(\sqrt{n}+1)}{\sqrt{n^6-n+1}}$ E. $\sum_{n=1}^{\infty} \frac{2}{3n}$

13. (18 points) Short Answers. Put your answer in the space provided:

(a) $\frac{d}{dx}e^{\tan^{-1}(x)} = ?$ (a) $e^{\tan^{-1}x} \cdot \frac{1}{1+x^2}$

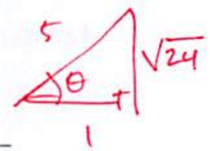
(b) Rewrite $\log_{(x^3)}(1+x^2)$ in terms of natural logarithms only. (b) $\frac{\ln(1+x^2)}{\ln x^3}$

$\log_a x = \frac{\ln x}{\ln a}$

(c) If $\theta = \sec^{-1}(5), \sin(2\theta) = ?$ (Your answer should be a number.) (c) $\frac{8\sqrt{6}}{25}$

$\sec\theta = 5$

$\frac{hyp}{adj}$



$\sin 2\theta = 2\sin\theta\cos\theta = 2 \cdot \frac{\sqrt{24}}{5} \cdot \frac{1}{5} = \frac{8\sqrt{6}}{25}$

(d) $\lim_{n\to\infty} \sqrt[n]{2n} =$ (d) 1

$\ln \sqrt[n]{2n} = \frac{\ln 2n}{n} \to 0 = L$ so $e^L = 1$.

(e) Rewrite $1.4144144\overline{144}\ldots$ in the form $\frac{P}{Q}$ where P and Q are integers. (e) $\frac{14}{10} + \frac{144/1000}{1 - 1/1000}$

$0.0144 + 0.0000144 + \cdots$

$r = \frac{1}{1000}$

$\frac{144}{10000} + \frac{144}{10000000} + \cdots$ first term $= \frac{144}{10000}$

(f) $\lim_{n\to\infty} (\frac{2n-1}{2n})^n = ?$ (f) $e^{-1/2}$

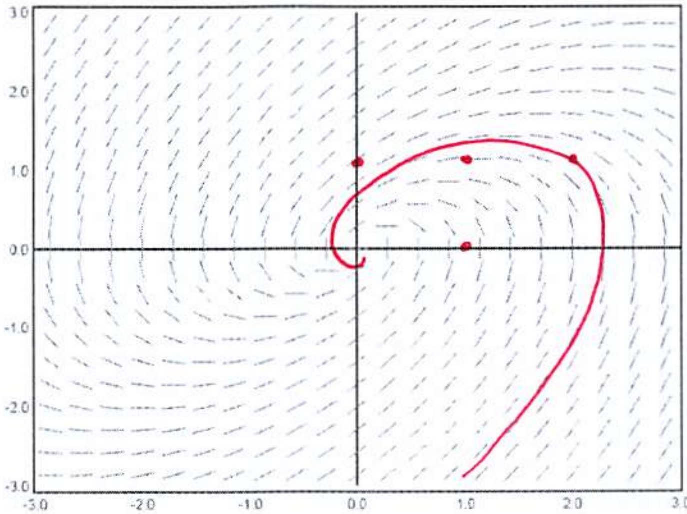
$= \lim_{n\to\infty} \left(1 + \frac{-1/2}{n}\right)^n$

The problems on this page are also short answer. You need not show work.

14. (10 points) (a) For the pictured slope field, circle the differential equation for which it is most likely the slope field:

A. $\frac{dy}{dx} = (y - x)/y$ B. $\frac{dy}{dx} = x + y$ C. $\frac{dy}{dx} = (y - x)/x$ D. $\frac{dy}{dx} = x/y$

	(0,1)	(1,0)	(1,1)
A	1	und.	0
~~B~~	1	1	2
~~C~~	und	-1	0
~~D~~	0	und	1



(b) On the same picture, sketch the solution to this equation that passes through the point $(2, 1)$

15. (7 points) Write out the *form* of a partial fractions decomposition for

$$\frac{x^2}{(x+1)(x-2)^2(x^2+1)^2}$$

(In other words, the very first step in a partial fractions decomposition.) Do NOT actually find any coefficients!!

$$\frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x-2} + \frac{Dx+E}{x^2+1} + \frac{Fx+G}{(x^2+1)^2}$$

(This page intentionally left blank. Use for scratch work or to finish a solution from earlier in the exam.)

Formula sheet

- Derivatives of inverse trigonometric functions.

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}\cot^{-1}(x) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}\sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\csc^{-1}(x) = -\frac{1}{|x|\sqrt{x^2-1}}$$

- Trigonometric identities.

$$\begin{aligned}
\sin^2 x + \cos^2 x &= 1 \\
1 + \tan^2 x &= \sec^2 x \\
1 + \cot^2 x &= \csc^2 x \\
\sin^2 x &= \frac{1}{2}(1 - \cos(2x)) \\
\cos^2 x &= \frac{1}{2}(1 + \cos(2x)) \\
\sin x \cos x &= \frac{1}{2}\sin(2x) \\
\sin x \sin y &= \frac{1}{2}\cos(x-y) - \frac{1}{2}\cos(x+y) \\
\cos x \cos y &= \frac{1}{2}\cos(x-y) + \frac{1}{2}\cos(x+y) \\
\sin x \cos y &= \frac{1}{2}\sin(x-y) + \frac{1}{2}\sin(x+y)
\end{aligned}$$

- Integrals of trigonometric functions.

$$\begin{aligned}
\int \tan x \, dx &= \ln|\sec x| + C \\
\int \cot x \, dx &= \ln|\sin x| + C \\
\int \sec x \, dx &= \ln|\sec x + \tan x| + C \\
\int \csc x \, dx &= -\ln|\csc x + \cot x| + C
\end{aligned}$$

- Trapezoidal Rule and Simpson's Rule.

$$T = \frac{\Delta x}{2}\Big(y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n\Big)$$
$$S = \frac{\Delta x}{3}\Big(y_0 + 4y_1 + 2y_2 + 4y_3 + \ldots + 2y_{n-2} + 4y_{n-1} + y_n\Big)$$

- Error estimates for Trapezoidal Rule and Simpson's Rule.

$$|E_T| \leq \frac{M(b-a)^3}{12n^2}, \quad \text{where } |f''(x)| \leq M \text{ for all } x \text{ in } [a,b]$$
$$|E_S| \leq \frac{M(b-a)^5}{180n^4}, \quad \text{where } |f^{(4)}(x)| \leq M \text{ for all } x \text{ in } [a,b]$$

- Famous Maclaurin series.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad (R = \infty)$$
$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \qquad (R = \infty)$$
$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \qquad (R = \infty)$$
$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n} \qquad (R = 1)$$
$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \qquad (R = 1)$$

- Error estimate for approximations by Taylor polynomials.

$$|R_n(x)| \leq \frac{M|x-a|^{n+1}}{(n+1)!},$$

where $|f^{(n+1)}(t)| \leq M$ for all t between a and x.