Math 242 Final Spring 2018

Name: ______________________________

(Please circle your section)

Section 1, MWF 9:30-10:20, Piper Harron

. Section 2, MWF 9:30-10:20, Piper Harron

Section 3, MWF 11:30-12:20, Jeffrey Lyons

Section 4, MWF 11:30-12:20, Jeffrey Lyons

Section 5, TR 9:00-10:15, Luca Candelori

Section 6, TR 9:00-10:15, Luca Candelori

Section 7, TR 12:00-1:15, Luca Candelori

Section 8, TR 12:00-1:15, Luca Candelori

Section 9, TR 1:30-2:45, Yohsuke Watanabe

Section 10, TR 1:30-2:45, Yohsuke Watanabe

Page	Points	Score
2	20	
3	30	
4	10	
5	15	
6	15	
7	25	
8	25	
9	15	
10	45	
12	15	
13	50	
Total:	265	

- You may *not* use notes or calculators on the test.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work and make clear what your final solution is (e.g. by drawing a box around it), except for the "short-answer" problems at the end.
- If you run out of room on a problem, continue on one of the blank pages after Page 13 and indicate clearly that you are doing this.
- On problems with choices (like Problem 12 do not do more parts than we ask you to, or you might be penalized!
- The last two pages are a formula sheet. You are welcome to remove this from the exam.
- **Don't forget to put your name on the paper!**
- Good luck!

1. (10 points) Find $f'(x)$ where $f(x) = \dfrac{2^x}{x^3\sqrt{1+x^2}}$.
   (Hint: use logarithmic differentiation.)

2. (10 points) $\lim_{t \to 0} \dfrac{\cos(t) - \cos(3t)}{t^2} = ?$

3. (30 points) Evaluate the integrals (on this page and the next).

(a) $\int \frac{\sin x dx}{2 + \cos x}$

(b) $\int z \tan^{-1}(z) dz = ?$

(c) $\int \sin^3 x \cos^2 x dx = ?$

4. (10 points) Estimate $\int_1^3 \frac{x}{1+x^2} dx$ using Simpson's rule with $n = 4$. **Don't simplify your answer!**

5. (15 points) Evaluate the integral: $$\int \frac{-2x+3}{(x+1)(x^2-2x+2)}dx$$

6. (15 points) Evaluate the integral: $\displaystyle\int \frac{\sqrt{x^2-9}}{x^3}dx$

7. (15 points) Evaluate, or show diverges: $\displaystyle\int_1^\infty \frac{e^{-1/x}}{x^2}dx$

8. (10 points) Set up a partial fractions decomposition for

$$\frac{x^2-1}{(x^2-4)(2x+1)^2(x^2+x+1)}$$

Do **not** find the relevant constants, just write down a sum of simple rational functions with unknown coefficients whose sum equals the rational function above.

9. (15 points) Test $\sum_{n=2}^{\infty} \frac{(-1)^n}{-1+\sqrt{n}}$ for (a) absolute and (b) conditional convergence. (c) How many terms would we need to compute the sum with an error of at most $\frac{1}{99}$?

10. (10 points) Test ONLY ONE of the following two series for convergence. **Cross out the one you do not want graded**:

$(a)\ \sum_{n=0}^{\infty} \frac{7^n}{n!}$ $\qquad$ $(b)\ \sum_{n=1}^{\infty} \left(\frac{2n^2+1}{3n^2-1}\right)^n$

11. (15 points) Find the **radius** and **interval** of convergence for $\sum_{n=0}^{\infty} \frac{3n^2}{e^n} x^n$

12. (45 points) Solve THREE of the following FOUR problems. DO NOT DO ALL FOUR!. Cross out the one you do **not** want graded.

(Continue your answers on the next page.)

(a) Solve the initial value problem: $y'' - 4y' + 4y = 0; \quad y(0) = 1, y'(0) = 0$

(b) Find the general solution to: $\frac{dy}{dx} = -y\ln(x)$

(c) Find the general solution to: $t\frac{dy}{dt} + 2y = t^3$

(d) In a certain culture of bacteria, the number of bacteria increased sixfold in 10 hours. How long did it take for their number to double?

13. (15 points) Find ONE of the following TWO series. DO NOT DO BOTH! Cross out the one you do **not** want graded.

(a) Find the Taylor series for $f(x) = \ln(x)$ at $x = e$. (Note: the first 5 terms will get you most of the credit)

(b) Find the MacLaurin series for $f(x) = \dfrac{1-x}{1+x}$. (Hint: what is the series for $\frac{1}{1+x}$?)

The problems on this page are short answer. You need not show work. They should all be *very quick*, if not you are doing it wrong!

14. (20 points) Circle all series that converge.

A. $\sum_{n=1}^{\infty}(\frac{n-3}{n})$ B. $\sum_{n=3}^{\infty}\frac{\ln(n)}{n^3}$ C. $\sum_{n=3}^{\infty}\frac{5n^3-3n}{n^2(n-2)(n^2+5)}$ D. $\sum_{n=0}^{\infty}(\sqrt{2})^n$ E. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$

15. (20 points) Short Answers. Put your answer in the space provided:

(a) $\log_2 64 = ?$ (a) ____________

(b) $\sum_{n=0}^{\infty}\frac{3^n}{2^{2n}} =$ (b) ____________

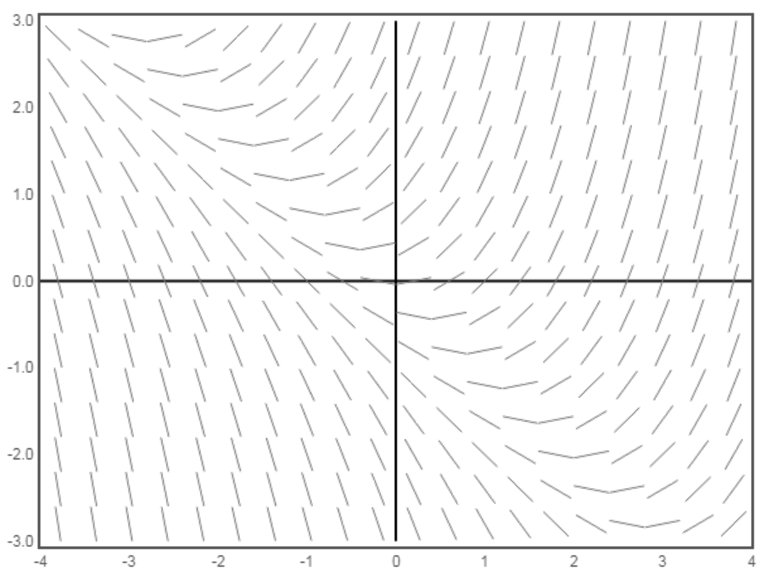
(c) $\sum_{k=1}^{\infty}(\frac{1}{k^2} - \frac{1}{k^2+2k+1}) =$ (c) ____________

(d) $\lim_{n\to\infty}(1+\frac{3}{n})^n =$ (d) ____________

(e) $\lim_{n\to\infty}(\sqrt[n]{n}) =$ (e) ____________

16. (10 points) (a) For the pictured slope field, circle the differential equation for which it is most likely the slope field:

A. $\frac{dy}{dx} = 2xy$ B. $\frac{dy}{dx} = x + y$ C. $\frac{dy}{dx} = 2y/x$ D. $\frac{dy}{dx} = 2x/y$



(b) On the same picture, sketch the solution to this equation that passes through the point $(1, 1)$

(This page intentionally left blank. Use for scratch work or to finish a solution from earlier in the exam.)

Formula sheet

- Derivatives of inverse trigonometric functions.

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}\cot^{-1}(x) = -\frac{1}{1+x^2}$$
$$\frac{d}{dx}\sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\csc^{-1}(x) = -\frac{1}{|x|\sqrt{x^2-1}}$$

- Trigonometric identities.

$$\begin{aligned}
\sin^2 x + \cos^2 x &= 1 \\
1 + \tan^2 x &= \sec^2 x \\
1 + \cot^2 x &= \csc^2 x \\
\sin^2 x &= \frac{1}{2}(1 - \cos(2x)) \\
\cos^2 x &= \frac{1}{2}(1 + \cos(2x)) \\
\sin x \cos x &= \frac{1}{2}\sin(2x) \\
\sin x \sin y &= \frac{1}{2}\cos(x-y) - \frac{1}{2}\cos(x+y) \\
\cos x \cos y &= \frac{1}{2}\cos(x-y) + \frac{1}{2}\cos(x+y) \\
\sin x \cos y &= \frac{1}{2}\sin(x-y) + \frac{1}{2}\sin(x+y)
\end{aligned}$$

- Integrals of trigonometric functions.

$$\begin{aligned}
\int \tan x \; dx &= \ln|\sec x| + C \\
\int \cot x \; dx &= \ln|\sin x| + C \\
\int \sec x \; dx &= \ln|\sec x + \tan x| + C \\
\int \csc x \; dx &= -\ln|\csc x + \cot x| + C
\end{aligned}$$

- Trapezoidal Rule and Simpson's Rule.

$$T = \frac{\Delta x}{2}\Big(y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n\Big)$$
$$S = \frac{\Delta x}{3}\Big(y_0 + 4y_1 + 2y_2 + 4y_3 + \ldots + 2y_{n-2} + 4y_{n-1} + y_n\Big)$$

- Error estimates for Trapezoidal Rule and Simpson's Rule.

$$|E_T| \le \frac{M(b-a)^3}{12n^2}, \quad \text{where } |f''(x)| \le M \text{ for all } x \text{ in } [a,b]$$
$$|E_S| \le \frac{M(b-a)^5}{180n^4}, \quad \text{where } |f^{(4)}(x)| \le M \text{ for all } x \text{ in } [a,b]$$

- Famous Maclaurin series.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad (R = \infty)$$
$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \qquad (R = \infty)$$
$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \qquad (R = \infty)$$
$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n} \qquad (R = 1)$$
$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \qquad (R = 1)$$

- Error estimate for approximations by Taylor polynomials.

$$|R_n(x)| \le \frac{M|x-a|^{n+1}}{(n+1)!},$$

where $|f^{(n+1)}(t)| \le M$ for all t between a and x.