Name: Solutions. Section: 11 12 13

Evaluate the integral using a trigonometric substitution.

$$\int x^3\sqrt{4-x^2}\,dx$$

Pattern: $\sqrt{4-x^2}$

use $x = 2\cdot\sin\theta$

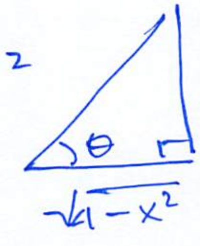
- $x^3 = 8\cdot\sin^3\theta$
- $dx = 2\cos\theta\,d\theta$
- $\sqrt{4-x^2} = \sqrt{4-4\sin^2\theta}$

$= 2\sqrt{\cos^2\theta}$

$= 2\cos\theta$

Triangle:

$\sin\theta = \dfrac{x}{2} = \dfrac{\text{opp}}{\text{hyp}}$



pyth.

$\Longrightarrow \cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{\sqrt{4-x^2}}{2}$

$$= \int 8\cdot\sin^3\theta\cdot 2\cos\theta\cdot 2\cos\theta\,d\theta$$

$$= 32\cdot\int \sin^3\theta\cdot\cos^2\theta\,d\theta$$

ODD POWER OF sin

use $u = \cos\theta$

$du = -\sin\theta\,d\theta$

$\sin^2\theta = 1-\cos^2\theta$

$= 1-u^2$

$$= 32\cdot\int \sin^2\theta\cdot\cos^2\theta\cdot\sin\theta\,d\theta$$

$$= 32\cdot\int (1-u^2)\,u^2\cdot(-du)$$

$$= 32\int u^4-u^2\,du$$

$$= 32\left[\frac{(\cos\theta)^5}{5} - \frac{(\cos\theta)^3}{3}\right]+C$$

$$= 32\left[\frac{1}{5}\left(\frac{\sqrt{4-x^2}}{2}\right)^5 - \frac{1}{3}\left(\frac{\sqrt{4-x^2}}{2}\right)^3\right]+C$$