Name: Solutions Section: 11 12 13

Determine if the following series converge or diverge; you may use techniques of geometric series, telescoping series, p-series, divergence test, integral test, alternating series test, and comparison tests.

1. $\sum_{n=1}^{\infty}(-1)^n \frac{n}{n^2+1}$ Alternating, try AST.

$b_n = \frac{n}{n^2+1}$

(1) b_n's are positive for $n \geq 1$

(2) b_n's are decreasing (larger denom)

(3) $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{n}{n^2+1} = \lim_{n\to\infty} \frac{1/n}{1+1/n^2} = \frac{0}{1+0} = 0$

$\therefore$ series converges

2. $\sum_{n=2}^{\infty} \frac{2^n}{5^n+3^n}$ Direct Comparison test:

$$\frac{2^n}{5^n+3^n} \leq \frac{2^n}{3^n+3^n} = \frac{2^n}{2\cdot 3^n} = \frac{1}{2}\cdot\left(\frac{2}{3}\right)^n$$

Since $\sum_{n=2}^{\infty} \frac{1}{2}\left(\frac{2}{3}\right)^n$ converges (geometric $r=\frac{2}{3}$, $|r|<1$)

then $\sum_{n=2}^{\infty} \frac{2^n}{5^n+3^n}$ converges