Name: Solutions Section: 11 12 13

Find the interval of convergence and radius of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{\sqrt{n}}{3^n}(x+2)^n.$$

Root test :

$$\rho = \lim_{n\to\infty} \sqrt[n]{\left| \frac{\sqrt{n}\,(x+2)^n}{3^n} \right|} = \lim_{n\to\infty} \sqrt[n]{\frac{\sqrt{n}\,|x+2|^n}{3^n}}$$

$$= \lim_{n\to\infty} \frac{(\sqrt[n]{n})^{1/2}\,|x+2|}{3} = \frac{|x+2|}{3}\cdot \lim_{n\to\infty}(\sqrt[n]{n})^{1/2}$$

$$= \frac{|x+2|}{3}\left(\lim_{n\to\infty}\sqrt[n]{n}\right)^{1/2}$$

$$= \frac{|x+2|}{3}\cdot 1$$

$$\rho < 1 \iff \frac{|x+2|}{3} < 1$$

$$\iff |x+2| < 3 \quad R$$

$$\iff -3 < x+2 < 3$$

$$\iff -5 < x < 1$$

Endpt $x=-5$: $\sum_{n=0}^{\infty} \frac{\sqrt{n}}{3^n}(-5+2)^n = \sum_{n=0}^{\infty}(-1)^n\cdot\sqrt{n}$

Diverges by Divergence test.

Endpt $x=1$: $\sum_{n=0}^{\infty} \frac{\sqrt{n}}{3^n}(1+2)^n = \sum_{n=0}^{\infty}\sqrt{n}$

Diverges by Divergence test.

I.O.C. $(-5,1)$, $R=3$