

Name: Solutions

Section: 11 12 13

1. Find the following values.

$$(a) \log_5(1/125) = x \iff 5^x = \frac{1}{125} = 5^{-3}$$

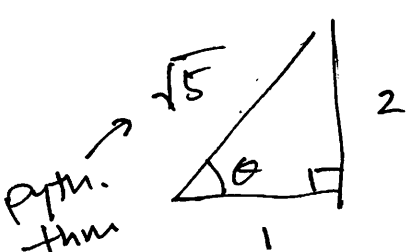
$$\iff x = -3$$

$$(b) \arcsin(1/2) = \theta \iff \sin \theta = \frac{1}{2}$$

$$\iff \theta = \frac{\pi}{6}$$

(c)  $\sin(\arctan(2))$  [Hint: Draw a triangle.]  $\arctan(2) = \theta \iff \tan \theta = 2 = \frac{\text{opp}}{\text{adj}}$

$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = \frac{2}{\sqrt{5}}$



2. Differentiate the following functions.

$$(a) f(x) = \log_{10}(\arccos x)$$

$$f'(x) = \frac{1}{(\arccos x) \cdot \ln(10)} \cdot \left[ -\frac{1}{\sqrt{1-x^2}} \right]$$

$$(b) y = \tan^{-1}(4x^2 - 1)$$

$$y' = \frac{1}{1 + (4x^2 - 1)^2} \cdot [4 \cdot 2x' - 0]$$

$$(c) f(x) = 2^{x^2} + \sec^{-1}(\ln x)$$

$$f'(x) = 2^{x^2} \cdot \ln(2) \cdot [2x] + \frac{1}{\ln x \sqrt{(\ln x)^2 - 1}} \cdot \frac{1}{x}$$

3. Evaluate the following integrals.

$$(a) \int \frac{3x}{1+x^4} dx = \int \frac{3x}{1+(x^2)^2} dx$$

$$\begin{array}{l} \text{u-sub} \\ u = x^2 \end{array}$$

$$du = 2x dx$$

$$= 3 \int \frac{\frac{1}{2} du}{1+u^2}$$

$$= \frac{3}{2} \int \frac{du}{1+u^2} = \frac{3}{2} \tan^{-1}(u) + C$$

$$= \frac{3}{2} \tan^{-1}(x^2) + C$$

$$(b) \int \frac{1}{\sqrt{3-2x-x^2}} dx \text{ [Hint: Complete the square!]}$$

Complete the square:

$$3-2x-x^2$$

$$= 3 - (x^2 + 2x)$$

$$= 3 - (x^2 + 2x + 1 - 1)$$

$$= 3 - ((x+1)^2 - 1)$$

$$= 4 - (x+1)^2$$

$$= \int \frac{1}{\sqrt{4 - (x+1)^2}} dx$$

$$= \int \frac{1}{\sqrt{4 - u^2}} du$$

$$= \int \frac{1}{2\sqrt{1 - (u/2)^2}} du$$

$$= \int \frac{1}{\sqrt{1 - v^2}} dv$$

$$= \arcsin(v) + C$$

$$= \arcsin\left(\frac{u}{2}\right) + C$$

$$= \arcsin\left(\frac{x+1}{2}\right) + C$$

u-sub

$$u = x+1$$

$$du = dx$$

u-sub

$$v = \frac{u}{2}$$

$$dv = \frac{1}{2} du$$