Name: Solutions Section: 11 12 13

1. Use integration by parts to evaluate the following integrals.

(a) $\int x \sin(3x)\, dx$

$u = x \qquad dv = \sin(3x)\,dx$

$du = dx \qquad v = -\frac{1}{3}\cos(3x)$

$$= -\frac{x}{3}\cos(3x) + \int \frac{1}{3}\cos(3x)\,dx$$

$$= -\frac{x}{3}\cos(3x) + \frac{1}{9}\sin(3x) + C$$

(b) $\int x^2 \ln x\, dx$

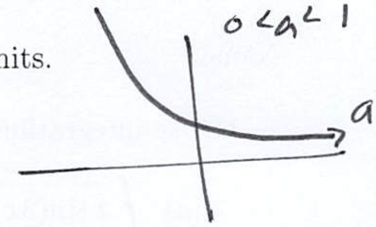
$u = \ln x \qquad dv = x^2$

$du = \frac{1}{x}dx \qquad v = \frac{x^3}{3}$

$$= \frac{x^3 \ln x}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x}\,dx$$

$$= \frac{x^3 \ln x}{3} - \frac{1}{3}\int x^2\,dx$$

$$= \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C$$



2. Use the methods we discussed in lecture to find the following limits.

(a) $\lim\limits_{x\to\infty} \dfrac{2^x+1}{3^x+1}$ Type $\frac{\infty}{\infty}$

$$\overset{l'H}{=} \lim_{x\to\infty} \frac{2^x \cdot \ln 2}{3^x \cdot \ln 3} = \frac{\ln 2}{\ln 3} \cdot \lim_{x\to\infty} \left(\frac{2}{3}\right)^x = 0.$$

(b) $\lim\limits_{x\to 0^+} (\ln x - \ln \sin x)$ Type $-\infty + \infty$

$$= \lim_{x\to 0^+} \ln\left(\frac{x}{\sin x}\right)$$

$$= \ln\left(\lim_{x\to 0^+} \frac{x}{\sin x}\right) \quad \text{Type } \frac{0}{0}$$

$$\overset{l'H}{=} \ln\left(\lim_{x\to 0^+} \frac{1}{\cos x}\right)$$

$$= \ln(1) = 0$$

(c) $\lim\limits_{x\to 0^+} \left(1+\dfrac{1}{x}\right)^x$

Set $L = \lim\limits_{x\to 0^+} \left(1+\frac{1}{x}\right)^x$ Type ∞^0

$$\Rightarrow \ln L = \lim_{x\to 0^+} x \cdot \ln\left(1+\frac{1}{x}\right) \quad \text{Type } 0\cdot\infty \text{ use } ab = \frac{a}{1/b}$$

$$\Rightarrow \ln L = \lim_{x\to 0^+} \frac{\ln\left(1+\frac{1}{x}\right)}{1/x} \quad \text{Type } \frac{\infty}{\infty}$$

$$\Rightarrow \ln L \overset{l'H}{=} \lim_{x\to 0^+} \frac{\frac{1}{(1+1/x)} \cdot (-1/x^2)}{(-1/x^2)}$$

$$= \lim_{x\to 0^+} \frac{1}{1+\underbrace{1/x}_{+\infty}}$$

$$= 0$$

$$\therefore \ln L = 0$$

$$\Rightarrow L = e^{\ln L} = e^0 = 1.$$