Name: Solutions Section: 11 12 13

1. $\int \tan^4 x \sec^4 x \, dx \longrightarrow = \int \tan^4 x \cdot \sec^2 x \cdot \sec^2 x \, dx$

EVEN SEC POWER

$u = \tan x$

$du = \sec^2 x \, dx$

$\sec^2 x = \tan^2 x + 1$

$= u^2 + 1$

$$= \int u^4 \cdot (u^2+1) \cdot du$$

$$= \int u^6 + u^4 \, du$$

$$= \frac{(\tan x)^7}{7} + \frac{(\tan x)^5}{5} + C$$

2. $\int x^5 \sqrt{1-x^2} \, dx \longrightarrow = \int \sin^5\theta \cdot \cos\theta \cdot \cos\theta \, d\theta$

Pattern: $\sqrt{1-x^2}$

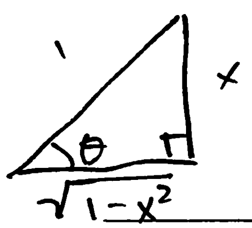
use $x = 1 \cdot \sin\theta$

- $x^5 = \sin^5\theta$
- $dx = \cos\theta \, d\theta$
- $\sqrt{1-x^2} = \sqrt{1-\sin^2\theta}$

$= \sqrt{\cos^2\theta}$

$= \cos\theta$

triangle: $\sin\theta = \frac{opp}{hyp} = \frac{x}{1}$



$\cos\theta = \frac{adj}{hyp}$

$= \sqrt{1-x^2}$

ODD SIN POWER

use $u = \cos\theta$

$du = -\sin\theta \, d\theta$

$\sin^2\theta = 1 - \cos^2\theta$

$= 1 - u^2$

$$= \int \sin^5\theta \cdot \cos^2\theta \, d\theta$$

$$= \int (\sin^2\theta)^2 \cdot \cos^2\theta \cdot \sin\theta \, d\theta$$

$$= \int (1-u^2)^2 \cdot u^2 \cdot (-du)$$

$$= -\int (1-2u^2+u^4) u^2 \, du$$

$$= -\int u^2 - 2u^4 + u^6 \, du$$

$$= -\left[\frac{(\cos\theta)^3}{3} - \frac{2}{5} \cdot \frac{(\cos\theta)^5}{} + \frac{(\cos\theta)^7}{7} \right] + C$$

$$= -\left[\frac{(\sqrt{1-x^2})^3}{3} - \frac{2}{5}(\sqrt{1-x^2})^5 + \frac{(\sqrt{1-x^2})^7}{7} \right] + C$$

3. $\int \frac{dx}{x^2\sqrt{9+x^2}}$ $\longrightarrow = \int \frac{\cancel{3}\sec^{\cancel{2}}\theta\, d\theta}{9\tan^2\theta \cdot \cancel{3}\cancel{\sec}\theta}$

Pattern: $\sqrt{9+x^2}$

use $x = 3\cdot\tan\theta$

· $dx = 3\sec^2\theta\, d\theta$

· $x^2 = 9\cdot\tan^2\theta$

· $\sqrt{9+x^2} = \sqrt{9+9\tan^2\theta}$

$= 3\sqrt{\sec^2\theta}$

$= 3\sec\theta$

$$= \frac{1}{9}\int \frac{\sec\theta}{\tan^2\theta}\, d\theta$$

convert to sines & cosines.

$$= \frac{1}{9}\int \frac{1/\cos\theta}{\sin^2\theta/\cos^2\theta}\, d\theta$$

$$= \frac{1}{9}\int \frac{\cos\theta}{\sin^2\theta}\, d\theta$$

<u>u-sub</u>

$u = \sin\theta \quad du = \cos\theta\, d\theta$

$$= \frac{1}{9}\int \frac{1}{u^2}\, du = \frac{1}{9}\int u^{-2}\, du$$

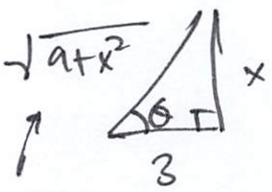
$$= -\frac{1}{9}\frac{1}{u} + C$$

$$= -\frac{1}{9}\frac{1}{\sin\theta} + C = -\frac{1}{9}\csc\theta + C$$

$$= -\frac{1}{9}\frac{\sqrt{9+x^2}}{x} + C$$

triangle!

$\tan\theta = \frac{x}{3} = \frac{opp}{adj}$



pyth.

$\therefore \csc\theta = \frac{hyp}{opp} = \frac{\sqrt{9+x^2}}{x}$