Name: Solutions Section: 11 12 13

1. Write the abstract partial fraction decomposition for the following functions.

$$\frac{x^3 - 19x + 4}{x^3(x^2+1)^2(6x-1)}$$

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+1} + \frac{Fx+G}{(x^2+1)^2} + \frac{H}{6x-1}$$

2. Evaluate $\int \frac{x^2-1}{x^2+x-12}\,dx$ $= \int 1 + \frac{-x+11}{x^2+x-12}\,dx$

deg(denom.) ≤ deg(numerator)
divide first.

$$= \int 1 + \frac{-x+11}{(x+4)(x-3)}\,dx$$

$$\begin{array}{r} 1 \\ x^2+x-12 \overline{) \; x^2 - 1} \\ -(x^2+x-12) \\ \hline -x+11 \end{array}$$

$$= \int 1 + \frac{-15/7}{x+4} + \frac{8/7}{x-3}\,dx$$

$$= x - \tfrac{15}{7}\ln|x+4| + \tfrac{8}{7}\ln|x-3| + C$$

$$\frac{-x+11}{(x+4)(x-3)} = \frac{A}{x+4} + \frac{B}{x-3}$$ clear fractions

$$-x+11 = A(x-3) + B(x+4)$$

$x=3$: $8 = A\cdot 0 + B\cdot 7 \Rightarrow B = 8/7$

$x=-4$: $15 = A(-7) + B\cdot 0 \Rightarrow A = -15/7$

3. Use the Trapezoidal Rule with $n = 4$ to approximate $\int_{-2}^{2} x^3 dx$.

$a = -2, \quad b = 2, \quad n = 4, \quad \Delta x = \frac{b-a}{n} = 1$

i	0	1	2	3	4
x_i	-2	-1	0	1	2
y_i	8	1	0	1	8
T_4 coef.	1	2	2	2	1

mult.

$\text{Sum} = 8 + 2 + 0 + 2 + 8 = 20$

$T_4 = \frac{\Delta x}{2} \cdot \text{Sum} = \frac{1}{2} \cdot 20 = 10.$

4. Find an integer n that would guarantee Simpson's Rule S_n to be within 10^{-4} of $\int_1^7 x^{5/2} dx$. You do not need to simplify your answer.

$f(x) = x^{5/2}$

$f'(x) = \frac{5}{2} x^{3/2}$

$f''(x) = \frac{15}{4} x^{1/2}$

$f'''(x) = \frac{15}{8} x^{-1/2}$

$\underbrace{f^{(4)}(x) = -\frac{15}{16} x^{-3/2}}_{\text{increases on } 1 \le x \le 7}$

$\Rightarrow |f^{(4)}(x)| = \left| -\frac{15}{16} x^{-3/2} \right| \le \frac{15}{16} \cdot 1^{-3/2} = 15/16$

we may use $K = 15/16$

want: $\frac{K(b-a)^5}{180 \cdot n^4} \le 10^{-4}$

$\frac{15/16\,(7-1)^5}{180 \cdot n^4} \le 10^{-4}$

Solve for n.

$\Rightarrow \frac{15/16\,(6)^5 \cdot 10^4}{180} \le n^4$

$\Rightarrow \sqrt[4]{\frac{15/16\,(6)^5 \cdot 10^4}{180}} \le n$

even n works.