Name: Sowtian Section: 11 12 13

1. Evaluate the following improper integrals.

(i) $\int_0^{\pi/2} \sec x \, dx$

$= \lim_{t \to \frac{\pi}{2}^-} \int_0^t \sec x \, dx$

$= \lim_{t \to \frac{\pi}{2}^-} \ln|\sec x + \tan x| \Big]_0^t$

$= \lim_{t \to \pi/2^-} \left[\ln|\sec t + \tan t| - 0 \right]$

$\ln|\sec t + \tan t| \to \infty$ as $t \to \frac{\pi}{2}^-$

$= \infty$

(ii) $\int_0^\infty \frac{1}{x^2+3x+2} \, dx$

$= \lim_{t \to \infty} \int_0^t \frac{1}{(x+1)(x+2)} \, dx$

$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

$1 = A(x+2) + B(x+1)$

$x = -2: \quad 1 = B(-1) \Rightarrow B = -1$

$x = -1: \quad 1 = A(1) \Rightarrow A = 1$

$\int \frac{1}{(x+1)(x+2)} \, dx = \int \frac{1}{x+1} + \frac{-1}{x+2} \, dx$

$= \ln|x+1| - \ln|x+2|$

$= \ln\left|\frac{x+1}{x+2}\right| + C$

$= \lim_{t \to \infty} \ln\left|\frac{x+1}{x+2}\right| \Big]_0^t$

$= \lim_{t \to \infty} \left[\ln\left|\frac{t+1}{t+2}\right| - \ln\left(\frac{1}{2}\right) \right]$

$= \ln\left| \lim_{t \to \infty} \frac{t+1}{t+2} \right| - \ln(1/2)$

$= \ln(1) - \ln(1/2)$

$= \ln(2)$

2. Write the first four terms of the following sequences and find the limit of each sequence.

(i) $\left\{\frac{2\sqrt{n}-1}{3+5n}\right\}_{n=0}^{\infty}$ $= \left\{-\frac{1}{3}, \frac{1}{8}, \frac{2\sqrt{2}-1}{13}, \frac{2\sqrt{3}-1}{18}, \ldots\right\}$

$$\lim_{n\to\infty} \frac{2\sqrt{n}-1}{3+5n} = \lim_{n\to\infty} \frac{2/\sqrt{n} - 1/n}{3/n + 5} = \frac{0-0}{0+5} = 0$$

(ii) $\left\{\left(1-\frac{3}{n}\right)^n\right\}_{n=1}^{\infty}$ $= \left\{(-2)^1, (-1/2)^2, 0, \left(\frac{1}{4}\right)^4, \ldots\right\}$

Common limit

$$\lim_{n\to\infty}\left(1-\frac{3}{n}\right)^n = \lim_{n\to\infty}\left(1+\frac{-3}{n}\right)^n = e^{-3}$$

(iii) $\left\{\frac{(-1)^n+2n}{n}\right\}_{n=1}^{\infty}$ $= \left\{1, \frac{5}{2}, \frac{5}{3}, \frac{9}{4}, \ldots\right\}$

Squeeze

$$-1+2n \le (-1)^n+2n \le 1+2n$$

$$\frac{-1+2n}{n} \le \frac{(-1)^n+2n}{n} \le \frac{1+2n}{n}$$

Since

$$\lim_{n\to\infty}\left(\frac{-1+2n}{n}\right) = 2 = \lim_{n\to\infty}\left(\frac{1+2n}{n}\right)$$

$$\Rightarrow \lim_{n\to\infty} \frac{(-1)^n+2n}{n} = 2.$$

(iv) $\left\{\left(-\frac{1}{2}\right)^n\right\}_{n=2}^{\infty}$

geometric sequence.

ratio $r = -1/2$

$|-1/2| < 1$

$$\Rightarrow \lim_{n\to\infty}\left(-\frac{1}{2}\right)^n = 0.$$

Common limit