Name: Solutions Section: 11 12 13

1. Find the first four terms of the $n$-th partial sums $\{s_n\}$ for the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$.

$$S_1 = \frac{1}{1} = 1$$

$$S_2 = S_1 + \left(-\frac{1}{2}\right) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$S_3 = S_2 + \frac{1}{3} = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

$$S_4 = S_3 - \frac{1}{4} = \frac{5}{6} - \frac{1}{4} = \frac{10-3}{12} = \frac{7}{12}$$

2. Find the sum of the following series

(a) $\sum_{n=1}^{\infty} 2^n 5^{1-n}$

geometric series

$$2^n \cdot 5^{1-n} = 5 \cdot \left(\frac{2}{5}\right)^n$$

ratio $= 2/5$

$|\text{ratio}| < 1$ converges

$$\text{Sum} = \frac{\text{first term}}{1 - \text{ratio}} = \frac{2^1 \cdot 5^{1-1}}{1 - 2/5} \cdot \frac{5}{5} = \frac{10}{5-2} = \frac{10}{3}$$

(b) $\sum_{n=1}^{\infty}\left(\frac{2}{\sqrt{n}} - \frac{2}{\sqrt{n+1}}\right)$ telescoping

$$S_n = \left(\frac{2}{\sqrt{1}} - \cancel{\frac{2}{\sqrt{2}}}\right) + \left(\cancel{\frac{2}{\sqrt{2}}} - \cancel{\frac{2}{\sqrt{3}}}\right) + \left(\cancel{\frac{2}{\sqrt{3}}} - \cancel{\frac{2}{\sqrt{4}}}\right) + \dots + \left(\cancel{\frac{2}{\sqrt{n}}} - \frac{2}{\sqrt{n+1}}\right)$$

$$= 2 - \frac{2}{\sqrt{n+1}}$$

$$\text{Sum} = \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(2 - \frac{2}{\sqrt{n+1}}\right) = 2 - 0 = 2$$

3. Determine if the following series converge or diverge. You may only use techniques of geometric series, telescoping series, the divergence test, $p$-series, and the integral test.

(a) $\displaystyle\sum_{n=1}^{\infty}\left(1+\frac{2}{n}\right)^n$ Divergence test!

common limit

$$\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^n = e^2 \neq 0$$

$\Longrightarrow$ series diverges.

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{2}{n^{3/2}} = 2\cdot\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$ p-series

$p = 3/2 > 1$

Series converges

(c) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\ln n}$ Integral test! $f(x)=\dfrac{1}{x\cdot\ln x}$

(1) pos. ✓
(2) cts. ✓
(3) dec. ✓

$$\int_2^{\infty}\frac{1}{x\cdot\ln x}dx = \lim_{t\to\infty}\int_2^t\frac{1}{x\cdot\ln x}dx = \lim_{t\to\infty}\int_{\ln 2}^{\ln t}\frac{du}{u}$$

u-sub
$u=\ln x$
$du = 1/x\,dx$

$$= \lim_{t\to\infty}\left[\ln\ln t - \ln\ln 2\right] = \infty$$

diverges.

$\Longrightarrow$ series diverges.

(d) $\displaystyle\sum_{n=0}^{\infty}\left(\frac{\pi}{3}\right)^n$

geometric

ratio $= \dfrac{\pi}{3}$

$|\text{ratio}| \geq 1$ $\Longrightarrow$ series diverges.