Name: Section: 11 12 13

1. Determine if the following series converge or diverge. You may only use techniques of geometric series, p-series, telescoping series, the divergence test, integral test, direct comparison, limit comparison, and alternating series test.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n + 3^n}$ DCT

$$\frac{1}{2^n+3^n} \le \frac{1}{2^n+2^n} = \frac{1}{2\cdot 2^n} = \frac{1}{2^{n+1}}$$

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^{n+1}}$ converges (geometric $r=\frac{1}{2}$, $|r|<1$)

$\therefore$ series converges

alternating

(b) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{\sqrt{n}}{n-1}$ try AST: $b_n = \dfrac{\sqrt{n}}{n-1}$

b_n's are positive and decreasing, and

$$\lim_{n\to\infty} \frac{\sqrt{n}}{n-1} = \lim_{n\to\infty} \frac{1/\sqrt{n}}{1-1/n} = \frac{0}{1-0} = 0.$$

$\therefore$ series converges

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{2n^3-n-11}$ behaves like $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2n^3} = \sum_{n=1}^{\infty} \frac{1}{2n^2}$ try LCT:

$$c = \lim_{n\to\infty} \frac{n-1}{2n^3-n-11} \cdot \frac{2n^3}{n} = \lim_{n\to\infty} \frac{2n^3}{2n^3-n-11} \cdot \frac{n-1}{n}$$

$$= \lim_{n\to\infty} \frac{2}{2-1/n^2-11/n^3} \cdot \frac{1-1/n}{\cancel{n/n}\ 1}$$

$$= \frac{2}{2-0-0} \cdot \frac{1-0}{1}$$

$= 1$

$0 < c < \infty \implies$ series converges since $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n^2}$ converges (p-series $p=2>1$)

alternating try AST

(d) $\sum_{n=2}^{\infty}(-1)^{n+1}\frac{\sqrt[3]{n}}{\sqrt[3]{n}-1}$, $b_n = \frac{\sqrt[3]{n}}{\sqrt[3]{n}-1}$ b_n's positive, dec? not obvious

note: $b_n = \frac{\sqrt[3]{n}}{\sqrt[3]{n}-1} = \frac{1}{1-1/\sqrt[3]{n}} \longrightarrow \frac{1}{1-0} = 1 \neq 0$

AST is inconclusive $\Longrightarrow$ try divergence test.

ODD terms : $\frac{\sqrt[3]{n}}{\sqrt[3]{n}-1} \longrightarrow 1$ } limit cannot exist so $\neq 0$

EVEN terms : $-\frac{\sqrt[3]{n}}{\sqrt[3]{n}-1} \longrightarrow -1$ } $\therefore$ series diverges

(e) $\sum_{n=3}^{\infty}\frac{1}{n(\ln n)^3}$ Integral test. $f(x) = \frac{1}{x(\ln x)^3}$ cts, pos, dec.

$$\int_3^{\infty}\frac{1}{x(\ln x)^3}\,dx = \int_{\ln 3}^{\infty}\frac{1}{u^3}\,du$$ converges (p-integral $p=3>1$)

$u = \ln x$

$du = \frac{1}{x}dx$

$u(3) = \ln 3$

$u \to \infty$ as $x \to \infty$

$\therefore$ series converges.

(f) $\sum_{n=1}^{\infty}(-1)^n\frac{3^n}{8^{n-1}} = \sum_{n=1}^{\infty} 8\cdot\left(-\frac{3}{8}\right)^n$

geometric $r = -\frac{3}{8}$

$|r| < 1$

$\therefore$ series converges