Name: Solutions Section: 11 12 13

1. Determine if the following series converge or diverge. You may use any technique.

(a) $\sum_{n=1}^{\infty} \frac{7\cos(n)}{n^2+1}$ ← random pos./neg. terms

try ACT

$\left|\frac{7\cos(n)}{n^2+1}\right| \le \frac{7}{n^2+1} \le \frac{7}{n^2}$ Since $\sum_{n=1}^{\infty} \frac{7}{n^2}$ converges (p-series, $p=2>1$), then $\sum_{n=1}^{\infty} \left|\frac{7\cos(n)}{n^2+1}\right|$ converges

$\Longrightarrow \sum_{n=1}^{\infty} \frac{7\cos(n)}{n^2+1}$ converges

(b) $\sum_{n=1}^{\infty} (-1)^n 2^{1/n}$ Divergence test!

odd terms : $(-1)^n 2^{1/n} = -2^{1/n} \longrightarrow -2^0 = -1$

even terms : $(-1)^n 2^{1/n} = 2^{1/n} \longrightarrow 2^0 = 1$

Hence, $\lim_{n\to\infty} (-1)^n 2^{1/n}$ DNE so $\neq 0$.

(c) $\sum_{n=3}^{\infty} \frac{(-5)^{n+1}}{2^{3n}} = \sum_{n=3}^{\infty} (-5)\cdot\frac{(-5)^n}{(8)^n} = \sum_{n=3}^{\infty} (-5)\left(\frac{-5}{8}\right)^n$

geometric $r = -\frac{5}{8}$, $|r|<1$

$\therefore$ series converges

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n!}{(n+2)!}$ factorials, try Ratio test

$$\rho = \lim_{n\to\infty} \left| \frac{2^{n+1}(n+1)!}{((n+1)+2)!} \cdot \frac{(n+2)!}{2^n \cdot n!} \right|$$

Since $\rho = 2 > 1$ the series diverges.

$$= \lim_{n\to\infty} \frac{\cancel{2^n}\cdot 2}{\cancel{2^n}} \cdot \frac{(n+1)\cdot \cancel{n!}}{\cancel{n!}} \cdot \frac{\cancel{(n+2)!}}{(n+3)\cancel{(n+2)!}}$$

$$= \lim_{n\to\infty} 2 \cdot \frac{n+1}{n+3} = 2 \cdot 1 = 2$$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$ powers of n, try Root test

$$\rho = \lim_{n\to\infty} \sqrt[n]{\left| \frac{(2n)^n}{n^{2n}} \right|}$$

$$= \lim_{n\to\infty} \sqrt[n]{\frac{(2n)^n}{(n^2)^n}}$$

$$= \lim_{n\to\infty} \frac{2n}{n^2} = \lim_{n\to\infty} \frac{2}{n} = 0$$

Since $\rho = 0 < 1$ series converges

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$ Integral test: $f(x) = \frac{\ln x}{x^2}$ pos, cts, dec.

IBP: $u = \ln x$ $\quad dv = \frac{1}{x^2}dx$

$du = \frac{1}{x}dx$ $\quad v = -\frac{1}{x}$

$$\int_1^{\infty} \frac{\ln x}{x^2}dx = \lim_{t\to\infty} \int_1^t \frac{\ln x}{x^2}dx$$

$$= \lim_{t\to\infty} \left[-\frac{\ln x}{x}\Big|_1^t + \int_1^t \frac{1}{x^2}dx \right]$$

$$= \lim_{t\to\infty} \left[-\frac{\ln t}{t} + \left[-\frac{1}{x}\right]_1^t \right] = \lim_{t\to\infty} \left[-\frac{\ln t}{t} - \frac{1}{t} + 1 \right] = 1$$

$\therefore$ series converges.