Name: Solutions. Section: 11 12 13

1. Find where the power series converges abosolutely, where it converges conditionally, and where it diverges. State the interval of convergence and the radius of convergence.

$$\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1},$$

Ratio test:

$$\rho = \lim_{n\to\infty}\left|\frac{(-1)^{n+1}x^{2(n+1)+1}}{2(n+1)+1}\cdot\frac{2n+1}{(-1)^n\cdot x^{2n+1}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{2n+1}{2n+3}\cdot\frac{x^{2n+1}\cdot x^2}{x^{2n+1}}\right| = x^2\cdot\lim_{n\to\infty}\frac{2n+1}{2n+3}$$

$$= x^2$$

$\rho<1 \iff x^2<1 \iff -1<x<1$ series converges abs.

Endpt. : $x=1$ $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{1^{2n+1}}{2n+1} = \sum_{n=0}^{\infty}(-1)^n\frac{1}{2n+1}$

$\displaystyle\sum|a_n| = \sum_{n=0}^{\infty}\frac{1}{2n+1}$ DCT : $\displaystyle\frac{1}{2n+2n} < \frac{1}{2n+1}$

$\displaystyle\frac{1}{4n} < \frac{1}{2n+1}$

Since $\displaystyle\sum_{n=0}^{\infty}\frac{1}{4n}$ diverges then $\displaystyle\sum_{n=0}^{\infty}\frac{1}{2n+1}$ diverges.

$b_n = \frac{1}{2n+1}$ pos, dec, $b_n\to 0$ series converges conditionally.

Endpt : $x=-1$ $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{(-1)^{2n+1}}{2n+1} = \sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{2n+1}$ converges conditionally (see above)

conv. abs: $-1<x<1$ diverges: $|x|>1$ $R=1$.

conv. cond: $x=\pm 1$ IOC : $-1\le x\le 1$

2. In lecture we showed that

$$\arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}, \qquad |x| \leq 1$$

(a) Find the sum of the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$

(b) Find a power series representation for $\frac{\arctan x}{x}$ using algebra.

(c) Use term by term integration to find a power series for $\int \frac{\arctan x}{x} dx$.

(a) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} = \arctan(1) = \pi/4$

(b) $\frac{1}{x} \arctan x = \frac{1}{x}\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots\right)$

$= 1 - \frac{x^2}{3} + \frac{x^4}{5} - \frac{x^6}{7} + \cdots$

(c) $\int \frac{\arctan(x)}{x} dx = \int 1 - \frac{x^2}{3} + \frac{x^4}{5} - \frac{x^6}{7} + \cdots dx$

$= C + x - \frac{x^3}{3^2} + \frac{x^5}{5^2} - \frac{x^7}{7^2} + \cdots$