Name: Solutions Section: 11 12 13

1. Find the Taylor polynomial of order 2 centered at $a = 8$ for $f(x) = x^{2/3}$.

$f(x) = x^{2/3} \implies f(8) = 8^{2/3} = 4$

$f'(x) = \frac{2}{3}x^{-1/3} \implies f'(8) = \frac{2}{3} \cdot 8^{-1/3} = \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$

$f''(x) = -\frac{2}{9}x^{-4/3} \implies f''(8) = -\frac{2}{9} \cdot 8^{-4/3} = -\frac{2}{9} \cdot \frac{1}{16} = -\frac{1}{72}$

$$T_2(x) = f(8) + f'(8)(x-8) + \frac{f''(8)}{2!}(x-8)^2$$

$$= 4 + \frac{1}{3}(x-8) + \frac{-1/72}{2!}(x-8)^2$$

2. Find a power series representation for $f(x) = 2^x$ by making the appropriate substitution in the Maclaurin series for e^x,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

$$2^x = e^{x \cdot \ln 2} = 1 + (x \cdot \ln 2) + \frac{(x \cdot \ln 2)^2}{2!} + \frac{(x \cdot \ln 2)^3}{3!} + \cdots$$

$$= 1 + \ln 2 \cdot x + \frac{(\ln 2)^2}{2!} \cdot x^2 + \frac{(\ln 2)^3}{3!} x^3 + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{(\ln 2)^n}{n!} \cdot x^n$$

3. Find the Maclaurin series for $f(x) = 2^x$ using the definition of a Maclaurin series.

$$f(x) = 2^x \implies f(0) = 2^0 = 1$$

$$f'(x) = 2^x \cdot \ln 2 \implies f'(0) = 2^0 \cdot \ln 2 = \ln 2$$

$$f''(x) = 2^x \cdot (\ln 2)^2 \implies f''(0) = 2^0 \cdot (\ln 2)^2 = (\ln 2)^2$$

$$\vdots$$

$$f^{(n)}(x) = 2^x \cdot (\ln 2)^n \implies f^{(n)}(0) = 2^0 (\ln 2)^n = (\ln 2)^n$$

$$\implies T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = \sum_{n=0}^{\infty} \frac{(\ln 2)^n}{n!} x^n$$