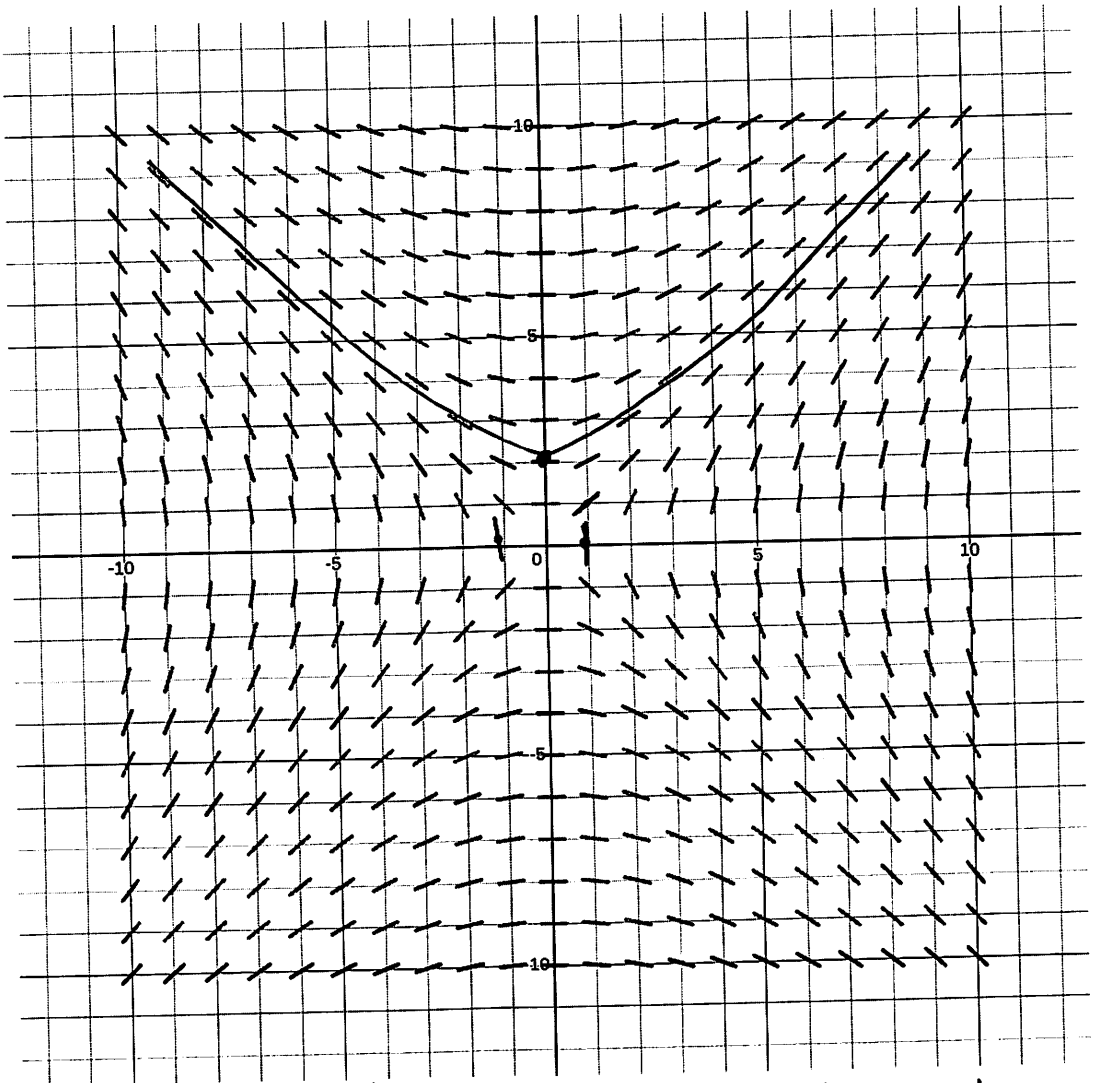
Name: Solutions

Section: 11 12 13

1. Match the DE with the direction field. Sketch the solution curve satisfying $y(0) = 2$.



(i) $y' = \frac{y}{x}$

(ii) $y' = \frac{x}{y}$

(iii) $y' = -\frac{y}{x}$

(iv) $y' = -\frac{x}{y}$

(x,y)	(i)	(ii)	(iii)	(iv)
$(1,0)$	0	undef. ✓	0	undef. ✓
$(1,1)$	1 ✓	1 ✓	-1	-1

this is the direction field for (ii).

2. Show that $y = \frac{1}{x}$ is a solution to the DE $x^3y''' + x^2y'' - 2xy' + 2y = 0$.

$y = x^{-1}$

$y' = -x^{-2}$

$y'' = 2x^{-3}$

$y''' = -6x^{-4}$

$x^3 y''' + x^2 y'' - 2xy' + 2y$

$= x^3(-6x^{-4}) + x^2(2x^{-3}) - 2x(-x^{-2}) + 2(x^{-1})$

$= -6x^{-1} + 2x^{-1} + 2x^{-1} + 2x^{-1}$

$= 0$ ✓

3. Find an upper bound for the error in using the approximation $x^{1/2} \approx T_2(x)$ (centered at $a = 4$) for $2 \le x \le 6$.

$f(x) = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f''(x) = -\frac{1}{4}x^{-3/2}$

$f'''(x) = \frac{3}{8}x^{-5/2}$

$= \underbrace{\frac{3}{8} \cdot \frac{1}{x^{5/2}}}$

decreasing on $2 \le x \le 6$

largest at $x = 2$

$|f'''(x)| \le \underbrace{\frac{3}{8}\frac{1}{2^{5/2}}}_{K}$

for $2 \le x \le 6 \iff |x-4| \le 2$

$\Longrightarrow |R_2(x)| \le \dfrac{\frac{3}{8} \cdot \frac{1}{2^{5/2}}}{3!}|x-4|^3$

$\le \dfrac{\frac{3}{8} \cdot \frac{1}{2^{5/2}}}{3!} \cdot 2^3$