Name: Solutions Section: 11 12 13

1. Solve the IVP

$$y' = \frac{xy^3}{\sqrt{1+x^2}}, \quad y(0) = -1$$

Separable.

u-sub
$u = 1+x^2$
$du = 2x\,dx$

$$\frac{dy}{y^3} = \frac{x}{\sqrt{1+x^2}}\,dx$$

$$\Longrightarrow \int \frac{dy}{y^3} = \int \frac{x}{\sqrt{1+x^2}}\,dx = \int u^{-1/2} \cdot \left(\tfrac{1}{2}du\right)$$

$$\Longrightarrow \frac{y^{-2}}{-2} = \frac{1}{2}\left(\frac{u^{1/2}}{1/2}\right) + C$$

$$\Longrightarrow -\frac{1}{2y^2} = \sqrt{1+x^2} + C$$

$$y(0) = -1 \Longrightarrow -\frac{1}{2} = \sqrt{1} + C \Longrightarrow C = -3/2$$

$$\Longrightarrow -\frac{1}{2y^2} = \sqrt{1+x^2} + (-3/2)$$

$$\Longrightarrow \frac{1}{y^2} = 3 - 2\sqrt{1+x^2}$$

$$\Longrightarrow y = \pm \frac{1}{\sqrt{3 - 2\sqrt{1+x^2}}}$$

root is always positive, since $y(0) = -1$ then we take the minus

$$y = -\frac{1}{\sqrt{3-2\sqrt{1+x^2}}}$$

2. Find the general solution to the ODE Linear.

$$xy' - 2y = x^5 \sin(2x)$$

std form : $y' - \frac{2}{x}y = x^4 \sin(2x)$

$p(x) = -\frac{2}{x}$ and $q(x) = x^4 \sin(2x)$

$\Longrightarrow P(x) = \int -\frac{2}{x}\,dx = -2\ln|x|$

$\Longrightarrow I(x) = e^{P(x)} = e^{-2\ln|x|} = |x|^{-2} = \frac{1}{x^2}$

Integrating factor

$\Longrightarrow y = \frac{1}{I(x)} \cdot \int q(x) \cdot I(x)\,dx.$

$= x^2 \cdot \int \frac{x^4 \cdot \sin(2x)}{x^2}\,dx = x^2 \cdot \underbrace{\int x^2 \sin(2x)\,dx}_{\text{IBP.}}$

x^2	$+$	$\sin(2x)$
$2x$	$-$	$-\frac{1}{2}\cos(2x)$
2	$+$	$-\frac{1}{4}\sin(2x)$
0		$\frac{1}{8}\cos(2x)$

$= x^2\left(-\frac{x^2}{2}\cos(2x) + \frac{x}{2}\sin(2x) + \frac{1}{4}\cos(2x) + C\right)$

$y = -\frac{x^4}{2}\cos(2x) + \frac{x^3}{2}\sin(2x) + \frac{x^2}{4}\cos(2x) + C \cdot x^2.$