Name: Solutions Section: 1 2

1. Use trigonometric substitution to evaluate $\int \frac{dx}{x^2\sqrt{9+x^2}}$

Form $\sqrt{a^2+x^2}$ use $x = a\cdot\tan\theta$

$x = 3\tan\theta$

$dx = 3\sec^2\theta\, d\theta$

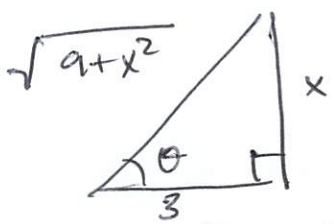
$\sqrt{9+x^2} = \sqrt{9+9\tan^2\theta}$

$= 3\sqrt{1+\tan^2\theta}$

$= 3\sqrt{\sec^2\theta}$

$= 3\sec\theta$

$x^2 = 9\tan^2\theta$



$\tan\theta = \frac{x}{3} = \frac{opp}{adj}$

$\csc\theta = \frac{hyp}{opp} = \frac{\sqrt{9+x^2}}{x}$

$$= \int \frac{3\sec^2\theta\, d\theta}{9\tan^2\theta \cdot 3\sec\theta}$$

$$= \frac{1}{9}\int \frac{\sec\theta}{\tan^2\theta}\, d\theta$$

$$= \frac{1}{9}\int \frac{\cos^2\theta}{\sin^2\theta}\cdot\frac{1}{\cos\theta}\, d\theta$$

$$= \frac{1}{9}\int \frac{1}{\sin^2\theta}\cos\theta\, d\theta$$

$u = \sin\theta$

$du = \cos\theta\, d\theta$

$$= \frac{1}{9}\int \frac{1}{u^2}\, du$$

$$= \frac{1}{9}\left(-\frac{1}{u}\right) + C$$

$$= -\frac{1}{9}\csc\theta + C$$

$$= -\frac{1}{9}\frac{\sqrt{9+x^2}}{x} + C$$

2. Evaluate the trigonometric integral $\int \tan^3 x \sec^4 x \, dx$

Even tan

$u = \tan\theta$

$du = \sec^2\theta \, d\theta$

$\sec^2\theta = 1 + \tan^2\theta$

$= 1 + u^2$

$$= \int \tan^3 x \cdot \sec^2 x \cdot \sec^2 x \, dx$$

$$= \int u^3 (1+u^2) \, du$$

$$= \int u^3 + u^5 \, du$$

$$= \frac{u^4}{4} + \frac{u^6}{6} + C = \frac{\tan^4\theta}{4} + \frac{\tan^6\theta}{6} + C$$

3. Use partial fractions to evaluate $\int \frac{x+3}{x^2+7x+6} \, dx$

Partial Fractions!

$$\frac{x+3}{x^2+7x+6} = \frac{x+3}{(x+6)(x+1)} = \frac{A}{x+6} + \frac{B}{x+1}$$

Clear fractions

$$\Rightarrow x+3 = A(x+1) + B(x+6)$$

Clever #6:

$x = -1$: $-1+3 = A\cdot 0 + B(5) \Rightarrow B = 2/5$

$x = -6$: $-6+3 = A(-5) + B\cdot 0 \Rightarrow A = 3/5$

$$\therefore \int \frac{x+3}{x^2+7x+6} \, dx = \int \frac{3/5}{x+6} + \frac{2/5}{x+1} \, dx$$

$$= 3/5 \ln|x+6| + 2/5 \ln|x+1| + C$$