Name: Solutions Section: 1 2

Determine if the following series converge or diverge. You may use any test. Show your work and clearly state which test you are using.

1. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}+1}$. Alternating Try AST $u_n = \frac{1}{\sqrt{n}+1}$

(1) $\frac{1}{\sqrt{n}+1} > 0$

(2) $\frac{1}{\sqrt{n+1}+1} \leq \frac{1}{\sqrt{n}+1}$

(3) $\frac{1}{\sqrt{n}+1} \to 0$

Series converges

2. $\sum_{n=1}^{\infty} \frac{1}{3^n+n}$. Try DCT

$\frac{1}{3^n+n} < \frac{1}{3^n}$

$\sum_{n=1}^{\infty} \frac{1}{3^n}$ geometric $|\text{ratio}| = |\frac{1}{3}| < 1$ converges

$\Rightarrow$ Series converges

3. $\sum_{n=2}^{\infty} (-1)^n \frac{n}{n-1}$. Alternating Try AST $u_n = \frac{n}{n-1}$

(1) $\frac{u}{n-1} > 0$

(2) $\frac{n}{n-1} = 1 + \frac{1}{n-1}$ is decreasing

(3) $\frac{n}{n-1} \to 1 \neq 0$ $\rightarrow$ try divergence test

evens $\to 1$

odds $\to -1$

So $\lim_{n\to\infty} (-1)^n \frac{n}{n-1}$ DNE so not 0.

Series diverges

4. $\sum_{n=2}^{\infty} \frac{(-5)^n}{n^2 7^n}$ ← Powers of n, try Root test

$$\rho = \lim_{n\to\infty} \sqrt[n]{\left|\frac{(-5)^n}{n^2 7^n}\right|}$$

Alt also works

$$= \lim_{n\to\infty} \frac{5}{(\sqrt[n]{n})^2 \cdot 7} = \frac{5}{1\cdot 7} = \frac{5}{7} < 1$$

Series converges

5. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$ Try DCT

$$\frac{1}{\sqrt{n}} < \frac{1}{\sqrt{n}-1}$$

$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$ p-series $p = 1/2 < 1$ diverges

$\Rightarrow \sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$ diverges

6. $\sum_{n=1}^{\infty} \frac{2n-1}{n^3-9}$. Behaves like $\sum_{n=1}^{\infty} \frac{2n}{n^3} = \sum_{n=1}^{\infty} \frac{2}{n^2}$ p-series $p=2>1$ converges

Try LCT

$$L = \lim_{n\to\infty} \frac{2n-1}{n^3-9} \cdot \frac{n^2}{2}$$

$$= \lim_{n\to\infty} \frac{2n^3-n^2}{2n^3-18} \cdot \frac{1/n^3}{1/n^3}$$

$$= \lim_{n\to\infty} \frac{2-1/n}{2-18/n^3} = \frac{2-0}{2-0} = 1$$

$0 < L < \infty$ then both series converge