Math 241 Homework 10 Solutions

Section 4.7

Problem 1. *Find an antiderivative for each function. Do as many as you can mentally. Check your answers by differentiation:*

(a) $2x$

(b) x^2

(c) $x^2 - 2x + 1$

Solution

(a) x^2 has derivative $2x$ so any equation of the form $x^2 + C$ is an antiderivative.

(b) $\dfrac{x^3}{3}$ has derivative x^2 so any equation of the form $\dfrac{x^3}{3} + C$ is an antiderivative

(c) $\dfrac{x^3}{3} - x^2 + x$ has derivative $x^2 - 2x + 1$ so any equation of the form $\dfrac{x^3}{3} - x^2 + x + C$ is an antiderivative.

□

Problem 3. *Find an antiderivative for each function. Do as many as you can mentally. Check your answers by differentiation:*

(a) $-3x^{-4}$

(b) x^{-4}

(c) $x^{-4} + 2x + 3$

Solution

(a) x^{-3} has derivative $-3x^{-4}$ so any equation of the form $x^{-3} + C$ is an antiderivative.

(b) $-\dfrac{1}{3}x^{-3}$ has derivative x^{-4} so any equation of the form $-\dfrac{1}{3}x^{-3} + C$ is an antiderivative.

(c) $-\dfrac{1}{3}x^{-3} + x^2 + 3x$ has derivative $x^{-4} + 2x + 3$ so any equation of the form $-\dfrac{1}{3}x^{-3} + x^2 + 3x + C$ is an antiderivative.

□

Problem 7. *Find an antiderivative for each function. Do as many as you can mentally. Check your answers by differentiation:*

(a) $\frac{3}{2}\sqrt{x}$

(b) $\frac{1}{2\sqrt{x}}$

(c) $\sqrt{x} + \frac{1}{\sqrt{x}}$

Solution

(a)

$$\frac{3}{2}\sqrt{x} = \frac{3}{2}x^{1/2}$$

So the general antiderivative can be found by

$$\frac{3}{2} \cdot \frac{x^{1/2+1}}{1/2+1} + C = \frac{3}{2} \cdot \frac{x^{3/2}}{3/2} + C = x^{3/2} + C$$

(b)

$$\frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-1/2}$$

So the general antiderivative can be found by

$$\frac{1}{2} \cdot \frac{x^{-1/2+1}}{-1/2+1} + C = \frac{1}{2} \cdot \frac{x^{1/2}}{1/2} + C = x^{1/2} + C$$

(c)

$$\sqrt{x} + \frac{1}{\sqrt{x}} = x^{1/2} + x^{-1/2}$$

So the general antiderivative can be found by

$$\frac{x^{1/2+1}}{1/2+1} + \frac{x^{-1/2+1}}{-1/2+1} + C = \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C = \frac{2}{3}x^{3/2} + 2x^{1/2} + C$$

□

Problem 11. *Find an antiderivative for each function. Do as many as you can mentally. Check your answers by differentiation:*

(a) $-\pi \sin \pi x$

(b) $3 \sin x$

(c) $\sin \pi x - 3 \sin 3x$

Solution

(a) The general antiderivative can be found by

$$-\pi\left(-\frac{\cos(\pi x)}{\pi}\right) + C = -\cos(\pi x) + C$$

(b) The general antiderivative can be found by

$$-3\cos x + C$$

(c) The general antiderivative can be found by

$$-\frac{\cos(\pi x)}{\pi} - 3\left(-\frac{\cos(3x)}{3}\right) + C = -\frac{\cos(\pi x)}{\pi} + \cos(3x) + C$$

$\square$

Problem 13. *Find an antiderivative for each function. Do as many as you can mentally. Check your answers by differentiation:*

(a) $\sec^2 x$

(b) $\dfrac{2}{3}\sec^2\dfrac{x}{3}$

(c) $-\sec^2\dfrac{3x}{2}$

Solution

(a) The general antiderivative is given by $\tan x + C$

(b) The general antiderivative is given by

$$\frac{2}{3}\cdot\frac{\tan x/3}{1/3} + C = 2\tan\frac{x}{3} + C$$

(c) The general antiderivative is given by

$$-\frac{\tan(3x/2)}{3/2} + C = -\frac{2}{3}\tan\frac{3x}{2} + C$$

$\square$

Problem 15. *Find an antiderivative for each function. Do as many as you can mentally. Check your answers by differentiation:*

(a) $\csc x\cot x$

(b) $-\csc 5x\cot 5x$

(c) $-\pi\csc\dfrac{\pi x}{2}\cot\dfrac{\pi x}{2}$

Solution

(a) The general antiderivative is given by $-\csc x + C$

(b) The general antiderivative is given by

$$-\frac{\csc(5x)}{5} + C$$

(c) The general antiderivative is given by

$$-\pi\left(-\frac{\csc(\pi x/2)}{\pi/2}\right) + C = \pi\cdot\frac{2}{\pi}\csc\frac{\pi x}{2} + C = 2\csc\frac{\pi x}{2} + C$$

$\square$

Problem 17. *Find the most general antiderivative or indefinite integral. Check your answers by differentiation:* $\int (x+1)\ dx$

Solution

$$\int (x+1)\ dx = \frac{x^{1+1}}{1+1} + \frac{x^{0+1}}{0+1} + C = \frac{x^2}{2} + x + C$$

Check:

$$\frac{d}{dx}\left(\frac{x^2}{2} + x + C\right) = \frac{2x}{2} + 1 = x + 1$$

$\square$

Problem 19. *Find the most general antiderivative or indefinite integral. Check your answers by differentiation:* $\int \left(3t^2 + \frac{t}{2}\right)\ dt$

Solution

$$\begin{aligned}\int \left(3t^2 + \frac{t}{2}\right)\ dt &= 3\cdot\frac{t^{2+1}}{2+1} + \frac{1}{2}\cdot\frac{t^{1+1}}{1+1} + C \\ &= 3\cdot\frac{t^3}{3} + \frac{1}{2}\cdot\frac{t^2}{2} + C \\ &= t^3 + \frac{t^2}{4} + C\end{aligned}$$

Check:

$$\frac{d}{dt}\left(t^3 + \frac{t^2}{4} + C\right) = 3t^2 + \frac{2t}{4} = 3t^2 + \frac{t}{2}$$

$\square$

Problem 31. *Find the most general antiderivative or indefinite integral. Check your answers by differentiation:* $\int 2x(1-x^{-3})\ dx$

Solution

$$\begin{aligned}\int 2x(1-x^{-3})\ dx &= \int (2x - 2x^{-2})\ dx \\ &= 2\cdot\frac{x^{1+1}}{1+1} - 2\cdot\frac{x^{-2+1}}{-2+1} + C \\ &= 2\cdot\frac{x^2}{2} - 2\cdot\frac{x^{-1}}{-1} + C \\ &= x^2 + \frac{2}{x} + C\end{aligned}$$

Check:

$$\frac{d}{dx}\left(x^2 + \frac{2}{x} + C\right) = 2x - \frac{2}{x^2} = 2x(1-x^{-3})$$

$\square$

Problem 33. *Find the most general antiderivative or indefinite integral. Check your answers by differentiation:* $\displaystyle\int \frac{t\sqrt{t}+\sqrt{t}}{t^2}\ dt$

Solution

$$\begin{aligned}
\int \frac{t\sqrt{t}+\sqrt{t}}{t^2}\ dt &= \int \left(\frac{t\sqrt{t}}{t^2}+\frac{\sqrt{t}}{t^2}\right)\ dt \\
&= \int \left(\frac{t^{3/2}}{t^2}+\frac{t^{1/2}}{t^2}\right)\ dt \\
&= \int \left(t^{3/2-2}+t^{1/2-2}\right)\ dt \\
&= \int \left(t^{-1/2}+t^{-3/2}\right)\ dt \\
&= \frac{t^{-1/2+1}}{-1/2+1}+\frac{t^{-3/2+1}}{-3/2+1}+C \\
&= \frac{t^{1/2}}{1/2}+\frac{t^{-1/2}}{-1/2}+C \\
&= 2t^{1/2}-\frac{2}{\sqrt{t}}+C
\end{aligned}$$

<u>Check:</u>

$$\frac{d}{dt}\left(2t^{1/2}-\frac{2}{\sqrt{t}}+C\right) = t^{-1/2}+t^{-3/2} = t^{-2}(t^{3/2}+t^{1/2}) = \frac{t\sqrt{t}+\sqrt{t}}{t^2}$$

□

Problem 35. $\displaystyle\int (-2\cos t)\ dt$

Solution

$$\int (-2\cos t)\ dt = -2\sin t + C$$

<u>Check:</u>

$$\frac{d}{dt}(-2\sin t + C) = -2\cos t$$

□

Problem 51. *Find the most general antiderivative or indefinite integral. Check your answers by differentiation:* $\int \cot^2 x \, dx$
(Hint: $1 + \cot^2 x = \csc^2 x$*)*

Solution

$$\int \cot^2 x \, dx \int (\csc^2 x - 1) \, dx = -\cot x - x + C$$

Check:

$$\frac{d}{dx}(-\cot x - x + C) = \csc^2 x - 1 = \cot^2 x$$

□

Problem 59. *Verify* $\int \frac{1}{(x+1)^2} \, dx = -\frac{1}{x+1} + C$ *by differentiation.*

Solution

$$\begin{aligned} \frac{d}{dx}\left(-\frac{1}{x+1} + C\right) &= \frac{d}{dx}(-(x+1)^{-1} + C) \\ &= (x+1)^{-2} \\ &= \frac{1}{(x+1)^2} \end{aligned}$$

□

Problem 63. *Right or wrong? Say which for each formula and give a brief reason for each answer.*

(a) $\int (2x+1)^2 \, dx = \frac{(2x+1)^3}{3} + C$

(b) $\int 3(2x+1)^2 \, dx = (2x+1)^3 + C$

(c) $\int 6(2x+1)^2 \, dx = (2x+1)^3 + C$

Solution

(a) Wrong.

$$\frac{d}{dx}\left(\frac{(2x+1)^3}{3} + C\right) = 2(2x+2)^2$$

(b) Wrong.

$$\frac{d}{dx}((2x+1)^3 + C) = 3(2x+1)^2(2) = 6(2x+1)^2$$

(c) Right. Same reasoning as (b).

□

Problem 95. *You are driving along a highway at a steady 60 mph (88 ft/sec) when you see an accident ahead and slam on the brakes. What constant deceleration is required to stop your car in 242 ft? To find out, carry out the following steps.*

(1) Solve the initial value problem

$$\textit{Differential equation: } \frac{d^2s}{dt^2} = -k \quad (k \text{ constant})$$

$$\textit{Initial conditions: } \frac{ds}{dt} = 88 \text{ and } s = 0 \text{ when } t = 0$$

Measuring time and distance from when brakes are applied.

(2) Find the value of t that makes $ds/dt = 0$. (The answer will involve k.)

(3) Find the value of k that makes $s = 242$ for the value of t you found in Step 2.

Solution

(1)

$$v(t) = \frac{ds}{dt} = \int -k\ dt = -kt + C$$

$$v(0) = 88 \to 88 = C \Rightarrow v(t) = -kt + 88$$

$$s(t) = \int (-kt + 88)\ dt = -\frac{kt^2}{2} + 88t + C$$

$$s(0) = 0 \Rightarrow C = 0 \Rightarrow s = -\frac{kt^2}{2} + 88t$$

(2)

$$v(t) = 0 \Leftrightarrow 0 = -kt + 88 \Leftrightarrow \boxed{t = \frac{88}{k} \text{ sec}}$$

(3) We want $s\left(\frac{88}{k}\right) = 242$

$$s\left(\frac{88}{k}\right) = 242 \Leftrightarrow -\frac{k}{2}\left(\frac{88}{k}\right)^2 + 88 \cdot \frac{88}{k} = 242$$

$$\Leftrightarrow -\frac{7744k}{2k^2} + \frac{7744}{k} = 242$$

$$\Leftrightarrow -\frac{3872}{k} + \frac{7744}{k} = 242$$

$$\Leftrightarrow \frac{3872}{k} = 242$$

$$\Leftrightarrow k = \frac{3872}{242} = \boxed{16}$$

□

Problem 96. *The State of Illinois Cycle Rider Safety Program requires riders to be able to brake from 30 mph (44ft/sec) to 0 in 45 ft. What constant deceleration does it take to do that?*

Solution Let $a = -k$ then

$$v = \int -k \; dt = kt + v_0 \text{ where } C = v_0 = v(0)$$

$$s = \int (kt + v_0) \; dt = -\frac{kt^2}{2} + v_0 t + s_0 \text{ where } C = s_0 = s(0)$$

We are given/assuming that $s(0) = 0$ and $v(0) = 44$ so we have

$$a(t) = -k, \; v(t) = -kt + 44, \text{ and } s(t) = -\frac{kt^2}{2} + 44t$$

When the car stops (say at $t = t_f$) we have $v = 0$ and

$$v(t_f) = 0 \Leftrightarrow -kt_f + 44 = 0 \Leftrightarrow k = \frac{44}{t_f}$$

Since we want to brake in 45 ft we want $s(t_f) = 45$ this gives

$$\begin{aligned} s(t_f) = 45 &\Leftrightarrow -\frac{44}{t_f} \cdot \frac{t_f^2}{2} + 44t_f = 45 \\ &\Leftrightarrow -22t_f + 44t_f = 45 \\ &\Leftrightarrow 22t_f = 45 \\ &\Leftrightarrow t_f = \frac{45}{22} \end{aligned}$$

Thus

$$k = \frac{44}{45/22} = \frac{44(22)}{45} = \frac{968}{45} \text{ ft/sec}$$

□

Problem 100. *For free fall near the surface of a planet where the acceleration due to gravity has a constant magnitude of g length-units/sec^2, Equation (1) in Exercise 99 takes the form*

$$s = -\frac{1}{2}gt^2 + \nu_0 t + s_0$$

where s is the body's height above the surface. The equation has a minus sign because the acceleration acts downward, in the direction of decreasing s. The velocity ν_0 is positive if the object is rising at time $t = 0$ and negative if the object is falling.

Instead of using the result of Exercise 99, you can derivate the above equation directly by solving an appropriate initial value problem. What initial value problem? Solve it to be sure you have the right one, explaining the solutions steps as you go along.

Solution

$$a(t) = -g,\ v(0) = v_0 \text{ and } s(0) = s_0$$

Then we have

$$v(t) = \int -g\ dt = -gt + C \text{ and } v(0) = C \Leftrightarrow C = v_0 \Rightarrow v(t) = -gt + v_0$$

$$s(t) = \int (-gt + v_0)\ dt = -\frac{gt^2}{2} + v_0 t + C \text{ and } s(0) = C \Leftrightarrow C = s_0 \Rightarrow s(t) = -\frac{gt^2}{2} + v_0 t + s_0$$

$\square$

Section 5.1

2.1 Area and Estimating with Finite Sums

Problem 1. *Use finite approximations to estimate the area under the graph of the function using*

(a) *a lower sum with two rectangles of equal width.*

(b) *a lower sum with four rectangles of equal width.*

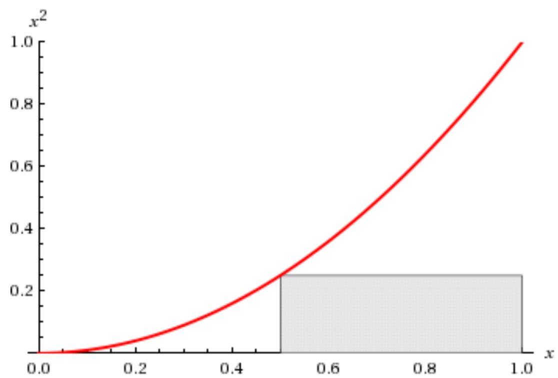
(c) *an upper sum with two rectangles of equal width.*

(d) *an upper sum with four rectangles of equal width.*

$$f(x) = x^2 \text{ between } x = 0 \text{ and } x = 1$$

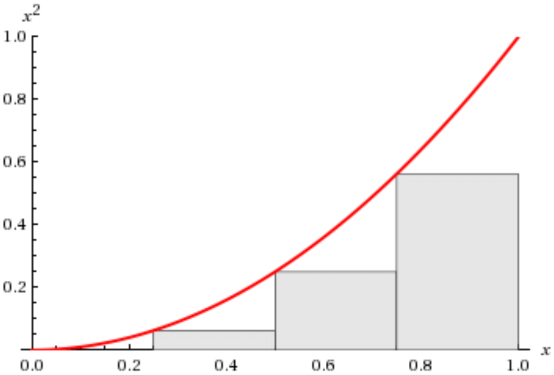
Solution

(a)



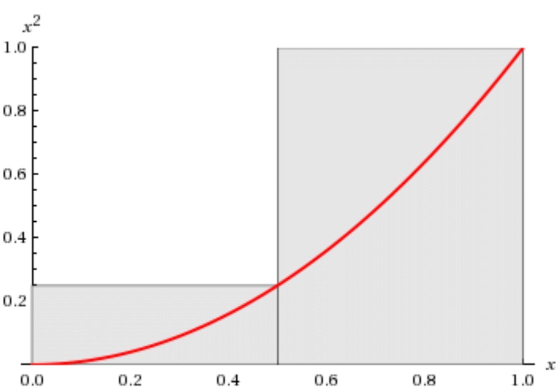
$$\begin{aligned} A &\approx \frac{1}{2}f(0) + \frac{1}{2}f\left(\frac{1}{2}\right) \\ &= \frac{1}{2}(0) + \frac{1}{2}\cdot\frac{1}{4} \\ &= \boxed{\frac{1}{8}} \\ &= 0.125 \end{aligned}$$

(b)



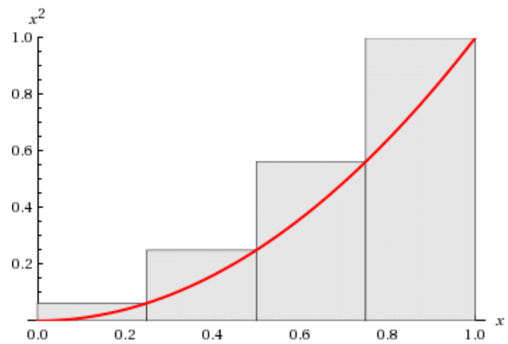
$$
\begin{aligned}
A &\approx \frac{1}{4}f(0) + \frac{1}{4}f\left(\frac{1}{4}\right) + \frac{1}{4}f\left(\frac{1}{2}\right) + \frac{1}{4}f\left(\frac{3}{4}\right) \\
&= \frac{1}{4}(0) + \frac{1}{4}\cdot\frac{1}{16} + \frac{1}{4}\cdot\frac{1}{4} + \frac{1}{4}\cdot\frac{9}{16} \\
&= \frac{1}{64} + \frac{1}{16} + \frac{9}{64} \\
&= \frac{1+4+9}{64} \\
&= \frac{14}{64} \\
&= \boxed{\frac{7}{32}} \\
&= 0.21875
\end{aligned}
$$

(c)



$$
\begin{aligned}
A &\approx \frac{1}{2} f\left(\frac{1}{2}\right) + \frac{1}{2} f(1) \\
&= \frac{1}{2} \cdot \frac{1}{4} + \frac{1}{2} \\
&= \frac{1}{8} + \frac{1}{2} \\
&= \frac{1}{8} + \frac{4}{8} \\
&= \boxed{\frac{5}{8}} \\
&= 0.625
\end{aligned}
$$

(d)



$$
\begin{aligned}
A &\approx \frac{1}{4} f\left(\frac{1}{4}\right) + \frac{1}{4} f\left(\frac{1}{2}\right) + \frac{1}{4} f\left(\frac{3}{4}\right) + \frac{1}{4} f(1) \\
&= \frac{1}{4} \cdot \frac{1}{16} + \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{9}{16} + \frac{1}{4} \\
&= \frac{1}{64} + \frac{1}{16} + \frac{9}{64} + \frac{1}{4} \\
&= \frac{1}{64} + \frac{4}{64} + \frac{9}{64} + \frac{16}{64} \\
&= \frac{30}{64} \\
&= \boxed{\frac{15}{32}} \\
&= 0.46875
\end{aligned}
$$

$\square$

Problem 3. *Use finite approximations to estimate the area under the graph of the function using*

(a) a lower sum with two rectangles of equal width.

(b) a lower sum with four rectangles of equal width.

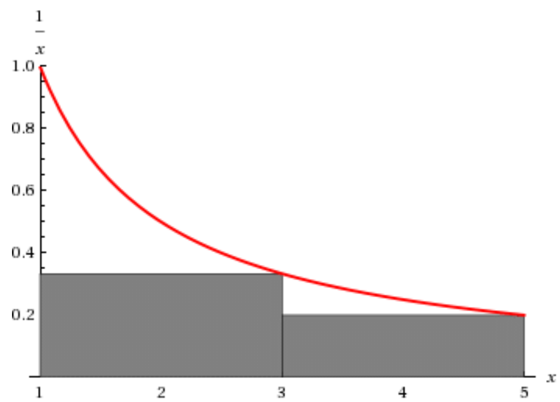
(c) an upper sum with two rectangles of equal width.

(d) an upper sum with four rectangles of equal width.

$$f(x) = \frac{1}{x} \text{ between } x = 1 \text{ and } x = 5$$

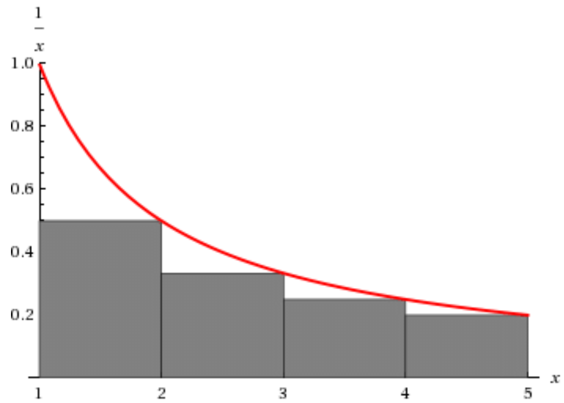
Solution f is decreasing so a lower sum uses right end points and an upper sum uses left end points.

(a)



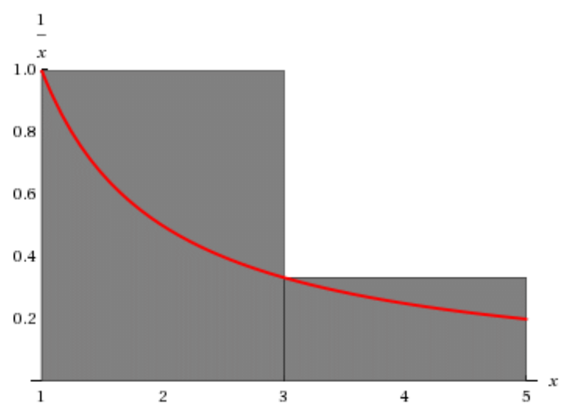
$$\begin{aligned} A &\approx \frac{1}{2}f(3) + \frac{1}{2}f(5) \\ &= 2 \cdot \frac{1}{3} + 2 \cdot \frac{1}{5} \\ &= \frac{2}{3} + \frac{2}{5} \\ &= \boxed{\frac{16}{15}} \end{aligned}$$

(b)



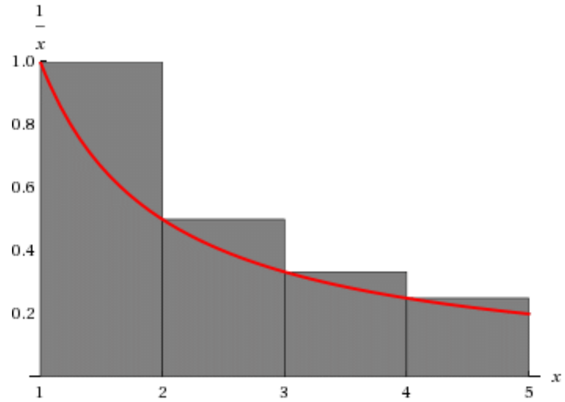
$$
\begin{aligned}
A &\approx f(2) + f(3) + f(4) + f(5) \\
&= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} \\
&= \frac{30}{60} + \frac{20}{60} + \frac{15}{60} + \frac{12}{60} \\
&= \boxed{\frac{77}{60}}
\end{aligned}
$$

(c)



$$
\begin{aligned}
A &\approx 2f(1) + 2f(3) \\
&= 2(1) + 2\frac{1}{3} \\
&= \boxed{\frac{8}{3}}
\end{aligned}
$$

(d)



$$
\begin{aligned}
A &\approx f(1) + f(2) + f(3) + f(4) \\
&= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \\
&= \frac{12}{12} + \frac{6}{12} + \frac{4}{12} + \frac{3}{12} \\
&= \boxed{\frac{25}{12}}
\end{aligned}
$$

$\square$

Problem 5. *Using rectangles whose height is given by the value of the function at the midpoint of the rectangle's base (the midpoint rule) estimate the area under the graphs of the following function, using first two and then four rectangles:* $f(x) = x^2$ *between* $x = 0$ *and* $x = 1$

Solution

Using 2 Rectangles:

$$\begin{aligned} A &\approx \frac{1}{2} f\left(\frac{1}{4}\right) + \frac{1}{2} f\left(\frac{3}{4}\right) \\ &= \frac{1}{2}\frac{1}{16} + \frac{1}{2}\frac{9}{16} \\ &= \frac{1}{32} + \frac{9}{32} \\ &= \frac{10}{32} \\ &= \boxed{\frac{5}{16} = 0.3125} \end{aligned}$$

Using 4 Rectangles:

$$\begin{aligned} A &\approx \frac{1}{4} f\left(\frac{1}{8}\right) + \frac{1}{4} f\left(\frac{3}{8}\right) + \frac{1}{4} f\left(\frac{5}{8}\right) + \frac{1}{4} f\left(\frac{7}{8}\right) \\ &= \frac{1}{4} \cdot \frac{1}{64} + \frac{1}{4} \cdot \frac{9}{64} + \frac{1}{4} \cdot \frac{25}{64} + \frac{1}{4} \cdot \frac{49}{64} \\ &= \frac{1}{256} + \frac{9}{256} + \frac{25}{256} + \frac{49}{256} \\ &= \frac{84}{256} \\ &= \boxed{\frac{21}{64} = 0.328125} \end{aligned}$$

$\square$

Problem 9. *The accompanying table shows the velocity of a model train engine moving along a track for 10 sec. Estimate the distance traveled by the engine using 10 subintervals of length 1 with:*

(a) left-endpoint values.

(b) right-endpoint values

Time (sec)	**Velocity (in./sec)**	**Time (sec)**	**Velocity (in./sec)**
0	0	6	11
1	12	7	6
2	22	8	2
3	10	9	6
4	5	10	0
5	13		

Solution

(a)

$$\begin{aligned}\text{Distance} &\approx f(0)+f(1)+f(2)+f(3)+f(4)+f(5)+f(6)+f(7)+f(8)+f(9)\\ &= 0+12+22+10+5+13+11+6+2+6\\ &= \boxed{87 \text{ inches}}\end{aligned}$$

(b)

$$\begin{aligned}\text{Distance} &\approx f(1)+f(2)+f(3)+f(4)+f(5)+f(6)+f(7)+f(8)+f(9)+f(10)\\ &= 12+22+10+5+13+11+6+2+6+0\\ &= \boxed{87 \text{ inches}}\end{aligned}$$

□

Problem 11. *You and a companion are about to drive a twisty stretch of dirt read in a car whose speedometer works but whose odometer (mileage counter) is broken. To find out how long this particular stretch of road is, you record the car's velocity at 10-sec intervals, with the results shown in the accompanying table. Estimate the length of the road using:*

(a) left-endpoint values.

(b) right-endpoint values.

Time (sec)	Velocity (converted to ft/sec) (30 mi/h = 44 ft/sec)	Time (sec)	Velocity (converted to ft/sec) (30 mi/h = 44 ft/sec)
0	0	70	15
10	44	80	22
20	15	90	35
30	35	100	44
40	30	110	30
50	44	120	35
60	35		

Solution

(a)

$$\begin{aligned}\text{Length} &= 10(f(0) + f(10) + f(20) + f(30) + f(40) + f(50) + f(60)\\ &\quad + f(70) + f(80) + f(90) + f(100) + f(110))\\ &= 10(0 + 44 + 15 + 35 + 30 + 44 + 35 + 15 + 22 + 35 + 44 + 30)\\ &= \boxed{3490 \text{ ft}}\end{aligned}$$

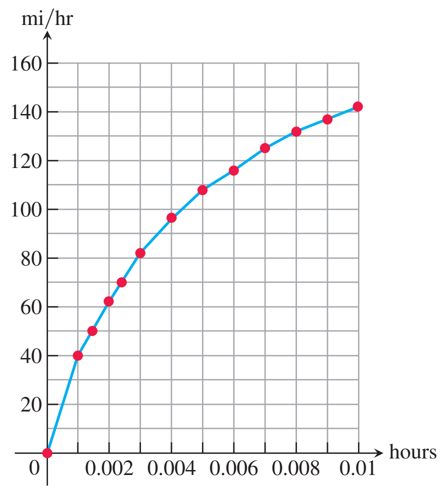
(b)

$$\begin{aligned}\text{Length} &= 10(f(10) + f(20) + f(30) + f(40) + f(50) + f(60)\\ &\quad + f(70) + f(80) + f(90) + f(100) + f(110) + f(120))\\ &= 10(44 + 15 + 35 + 30 + 44 + 35 + 15 + 22 + 35 + 44 + 30 + 35)\\ &= \boxed{3840 \text{ ft}}\end{aligned}$$

□

Problem 12. *The accompanying table gives data for the velocity of a vintage sports car accelerating from 0 to 142 mi/h in 36 sec (10 thousandths of an hour).*

Time (h)	Velocity (mi/h)	Time (h)	Velocity (mi/h)
0.0	0	0.006	116
0.001	40	0.007	125
0.002	62	0.008	132
0.003	82	0.009	137
0.004	96	0.010	142
0.005	108		



(a) *Use rectangles to estimate how far the car traveled during the 36 sec it took to reach 142 mi/h.*

(b) *Roughly how many seconds did it take the car to reach the halfway point? About how fast was the car going then?*

Solution

(a)

$$\begin{aligned}\text{Distance} &\approx 0.001(0+40+62+82+96+108+116+125+132+137+142)\\ &= 1.04 \text{ miles}\end{aligned}$$

(b) The halfway point would be 0.52 miles. so we are estimating until the sum of the velocities gets to 520 miles/hour.

$$40+62+82+96+108+116=504$$

So the halfway point occurs between 0.006 and 0.007 hours.

□

Problem 13. *An object is dropped straight down from a helicopter. The object falls faster and faster but its acceleration (rate of change of its velocity) decreases over time because of air resistance. The acceleration is measured in ft/sec^2 and recorded every second after the drop for 5 sec as shown*

t	0	1	2	3	4	5
a	32.00	19.41	11.77	7.14	4.33	2.63

(a) *Find an upper estimate for the speed when* $t = 5$.

(b) *Find a lower estimate for the speed when* $t = 5$.

(c) *Find an upper estimate for the distance fallen when* $t = 3$

Solution

(a) Since the acceleration is decreasing an upper estimate is found using left end points

$$\begin{aligned}\text{Speed} &\approx 32 + 19.41 + 11.77 + 7.14 + 4.33 \\ &= \boxed{74.65 \text{ ft/sec}}\end{aligned}$$

(b) A lower estimate is found using right end points

$$\begin{aligned}\text{Speed} &\approx 19.41 + 11.77 + 7.14 + 4.33 + 2.63 \\ &= \boxed{45.28 \text{ ft/sec}}\end{aligned}$$

(c) At time $t = 1$ velocity is upper estimated to be 32 ft/sec. At $t = 2$ velocity is upper estimated to be $32 + 19.41 = 51.41$ and at time 3 velocity is upper estimated to be $32 + 19.41 + 11.77 = 63.18$. Thus the distance can be upper estimated to be

$$32 + 51.41 + 63.18 = \boxed{146.59 \text{ ft}}$$

□

Problem 19. *Oil is leaking out of a tanker damaged at sea. The damage to the tanker is worsening as evidenced by the increased leakage each hour, recorded in the following table.*

Time (h)	0	1	2	3	4
Leakage (gal / h)	50	70	97	136	190

Time (h)	5	6	7	8
Leakage (gal / h)	265	369	516	720

(a) *Given an upper and lower estimate of the total quantity of oil that has escaped after 5 hours.*

(b) *Repeat part (a) for the quantity of oil that has escaped after 8 hours.*

(c) *The tanker continues to leak 720 gal/h after the first 8 hours. If the tanker originally contained 25,000 gal of oil, approximately how many more hours will elapse in the worst case before all oil has spilled? In the best case?*

Solution

(a) Since the leakage is increasing the upper estimate is given by right end points and the lower is given by left end points.

$$\begin{aligned}\text{Upper Estimate} &\approx 70 + 97 + 136 + 190 + 265 \\ &= \boxed{758 \text{ gallons}} \\ \text{Lower Estimate} &\approx 50 + 70 + 97 + 136 + 190 \\ &= \boxed{543 \text{ gallons}}\end{aligned}$$

(b)

$$\begin{aligned}\text{Upper Estimate} &\approx 70 + 97 + 136 + 190 + 265 + 369 + 516 + 720 \\ &= \boxed{2363 \text{ gallons}} \\ \text{Lower Estimate} &\approx 50 + 70 + 97 + 136 + 190 + 265 + 369 + 516 \\ &= \boxed{1693 \text{ gallons}}\end{aligned}$$

(c) In the worst case scenario, 2363 gallons have already escaped leaving 22,637 gallons left.

$$\frac{22637}{720} \approx \boxed{31.44 \text{ hours}}$$

So it should leak for around 32 more hours. In the best case scenario, 1693 gallons have escaped leaving 23,307 gallons left.

$$\frac{23307}{720} \approx \boxed{32.37 \text{ hours}}$$

□