ETHNOMATHEMATICS CURRICULUM TEXTBOOK

Lesson Plans in Symbolic Reasoning and Quantitative Literacy

Funded by the National Science Foundation, U.S. Department of Education Title III, Hawaiʻi Pacific Islands Campus Compact Oceanic EcosySTEM, and University of Hawaiʻi Student Equity, Excellence, and Diversity Office, with support from the University of Hawaiʻi - West Oʻahu
Ethnomathematics Curriculum Textbook:
Lesson Plans in Symbolic Reasoning and Quantitative Literacy
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**INTRODUCTION**

Dr. Linda H.L. Furuto, UHWO Mathematics Faculty

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**REFERENCES**
INTRODUCTION

Students, faculty, and staff engage in ethnomathematics research at Kalaupapa National Historical Park on the island of Moloka‘i.

Written by Dr. Linda H.L. Furuto

Overview

A number of studies show Hawai‘i’s reference to national and international populations. The Programme for International Student Assessment (PISA) is a triennial survey of the knowledge and skills of 15 year olds. It is the product of collaboration between participating countries and economies through the Organisation for Economic Cooperation and Development (OECD). More than 400,000 students in 65 countries and economies making up close to 90% of the world economy took part in PISA 2009. Assessment included data on student, community, and institutional factors that could help to explain differences in mathematics performance such as culture and family background. The U.S. currently ranks 27th out of 34 OECD member countries in mathematics, which is statistically significantly below the OECD average. The U.S. mean score is 287, and the OECD mean score average is 494 (PISA, 2009).

We know the U.S. has challenges to overcome, but some states are performing extremely well such as Massachusetts. Near the other end of the spectrum is Hawai‘i. The National Assessment of Educational Progress (NAEP) mathematics assessment is the largest nationally representative and continuing assessment and is administered periodically to 4th, 8th, and 12th graders. NAEP results serve as a common metric for all states and selected urban districts. In a state comparison of NAEP 8th grade mathematics average scaled scores from the 2011 assessment, Hawai‘i’s average scale score of 278 is significantly lower than the national average of 283. The percentage of students in Hawai‘i who performed at or above the NAEP proficient level was 40% in 2011, and Hawai‘i was ranked 44 out of 50 states (NCES, 2011).

To address these challenges, the first statewide Mathematics Summit, designed to facilitate discussion and action on raising mathematics achievement levels of Hawai‘i students to ensure college and career success, was held in 2008. This is a collaborative effort between the major stakeholders in the state, the University of Hawai‘i Systemwide Office, State of Hawai‘i Department of Education, and Hawai‘i P-20 Partnerships for Education. At that time, University of Hawai‘i Vice President of Academic Planning and Policy Linda Johnsrud stated, “The conceptual ability to do the process that comes with doing mathematics is paramount. We don’t need more human calculators; we need global citizens who think critically about the world around them and make meaningful connections with mathematics” (Johnsrud, L., Personal Communication, February 6, 2012).

The goals of the State of Hawai‘i are to: improve the mathematics pipeline leading to career and college ready mathematics, improve alignment of courses so that students transition smoothly between institutions and courses, and prepare more qualified and effective mathematics educators. Given the framework outlined at the international, national, and local levels, ethnomathematics was utilized as a tool to bridge research and practice to provide equitable, quality education.
Ethnomathematics

The term ethno describes “ingredients that make up the identity of a group: language, codes, values, beliefs, community, and class” (D’Ambrosio, 2001, p. 308). Mathematics expresses a “broad view which includes arithmetic, classifying, ordering, inferring, and modeling” (D’Ambrosio, 2001, p. 308). An understanding of these terms allows educators to expand their mathematics perceptions and more effectively instruct students in a growing school climate of diversity; and one that is particularly present in the State of Hawai’i.

Just as literacy has come to mean much more than reading and writing, mathematics must also be thought of as more than counting, calculating, sorting, and comparing. Today’s children are living in a civilization that functions by mathematically based technology and unprecedented means of communication. A goal of mathematics education should be to build upon students’ abilities to successfully use a diverse array of knowledge to solve problems and communicate their thinking as they gain an awareness of the capabilities and limitations of an interconnected global community (Ascher, 1987, 1991; Thomas, 1985, 1987). Many societies, including Native Hawaiian, did not have a formal system of writing until Western contact. As a result, the category “mathematics” is not necessarily found in traditional cultures. In Hawai’i, people performed complex calculations of trigonometric angles, algebraic wind speed, and geometrical star houses for centuries, yet there was no word for “mathematician” or “scientist.” While it is very important to learn about mathematics contained in textbooks, it is equally critical to recognize that mathematics is everywhere.

Ethnomathematics encourages us to witness and attempt to understand how mathematics continues to be adapted and used by people around the world. Students should be encouraged to construct personal mathematical understandings and be able to explain their work. When all students’ inventions, experiences, and applications of mathematics are realized and respected, they are given equal opportunity for access and achievement (Harvard University Achievement Gap Initiative, 2008; Kyselka, 1987; UCLA). Diversity, and one that is particularly present in the State of Hawai’i.

◊ Based on research and practicum, publish a textbook with lesson plans to be used by current and future mathematics teachers in the State of Hawai’i as resource materials, supplementary curriculum, and teacher training guides. The lesson plans cover core mathematics content in the areas of symbolic reasoning and quantitative literacy. These are among the primary areas mathematics teachers need to develop competency as deemed by the Hawai’i Teachers Standard Board.

The most successful mathematics education programs address issues of support and retention with a multi-faceted approach involving a variety of strategies that celebrate the diversity and heritage of students, and that was the aim of this project (Astin & Oseguera, 2005; Babayan, Finney, Kilonsky, & Thompson, 1987; Bok, 2006; National Center for Public Policy and Higher Education, 2006). This textbook draws on the unique pedagogies, values, and mathematics of our sacred islands.

Lesson Plans

The Ethnomathematics Summer Institute involved 50 University of Hawai’i mathematics faculty, staff, and students grouped into teams to focus on the core mathematics education areas of symbolic reasoning and quantitative literacy. Institute scholars created and carried out individual projects, based upon selected topics to prepare for advanced-level mathematics.

Through classroom learning, mathematics field studies, and real-world applications of mathematics, institute scholars produced the following lesson plans contained in this textbook: Micronesian Sand Drawings, Traditional Hawaiian Quilting, Mathematics and Native Hawaiian Plants, Geometric Equations in Hawai’i, Hawai’i’s Kalo Culture, Musical Frequency of the Ukulele, Vectors and Navigating a Voyaging Canoe, The Hawaiian Star Compass and the Unit Circle, Seafaring Traditions and Mathematical Applications and Derivatives and Integrals in Art Imaging. Each lesson plan is formatted with four parts to assist with implementation: (1) introduction and history, (2) goal of lesson plan, (3) methodology, and (4) conclusion. The lesson plans reflect the culture and traditions of the communities we serve, and informs and directs our mathematics education efforts.
Mathematics Field Studies and Partnerships

In conjunction with the Polynesian Voyaging Society, Kalaupapa National Historical Park, Hawai'i Institute of Marine Biology, Mokaua Island Fishing Village, Hawai'i Council of Teachers of Mathematics, and State of Hawai'i Department of Education, institute scholars participated in mathematics field studies throughout the Hawaiian islands.

At the Hawai'i Institute of Marine Biology's Coconut Island in Kāne'ohe Bay, students learned about intersections of natural resources, environmental conservation, tropical marine science, and mathematics through a bioacoustics laboratory. Mokaua Island is the site of O'ahu's last Hawaiian fishing village, and one of only two left in Hawai'i where hundreds of villages thrived in pre-European times. Through an experiential learning environment, students helped restore the fishing village and learned about mathematical skills and the practice of fishing, seafaring, and maintaining healthy viable oceans. The Polynesian Voyaging Society helped University of Hawai'i mathematics faculty and students undertake voyages of discovery with a legacy of ocean exploration as its foundation. We respect, learn from, and perpetuate heritage through practice, and in turn promote learning which integrates voyaging experiences and values into quality mathematics education. Through sailing on the voyaging canoes by traditional, celestial navigation (i.e., sun, moon, stars, tides, etc.), we witnessed how mathematics is a tool to learn about real-world applications, and share knowledge. Finally, Kalaupapa, Moloka'i was once a leprosy community in isolation, and now serves as a place for education and contemplation. Past suffering has given way to personal pride about accomplishments made in the face of great adversity. Kalaupapa is a place where the land has the power to heal because of its human history, natural history, and stunning physical beauty. Through first-hand mathematical experiences on land and sea, we came to understand how these themes impact present and future generations.

The Ethnomathematics Summer Institute involved collaboration between community-based organizations and research institutions, and was funded by the National Science Foundation, U.S. Department of Education Title III, Hawai'i Pacific Islands Campus Compact Oceanic EcosysSTEM, and University of Hawai'i Student Equity, Excellence, and Diversity Office. A very special mahalo to Mellissa Lochman and Sharla Hanaoka for their support with textbook design, layout, and publication. This is our contribution to our island home.
References


Section One: Elementary Level
Part 1: Introduction and History

Through the Ethnomathematics Summer Institute, I personally came to realize that life is a dynamic educational institution in itself. Languages, behaviors, knowledge, skills, and talents are acquired and developed naturally, as one grows. Ubiratan D’Ambrosio illustrated in his book, Ethnomathematics: Link between Traditions and Modernity, that “mathematics have developed as a result of the interaction of humans with the environment” (D’Ambrosio, 2001, p. 4). However, as children reach school age, their ways of learning change. They move from the comfort of their home into a confined classroom, and from hands-on learning to abstract learning.

In school, skills are categorized into separate subjects such as language arts, science, social studies, and mathematics. Unfortunately, the name of some subjects carry a psychological stigma leading students to treat them with hostility. From personal experience, mathematics is the most commonly dreaded subject among students. Perhaps adding to the confusion is the shift from the traditional use of tangible objects, a physical representation of the theoretical knowledge, to the more abstract model of mathematics. To ensure that learning transitions to this new model, relevant approaches must also be designed for the classroom. Different theories have been proposed to achieve this. According to Piaget, young children “learn about and understand the world only by physically manipulating objects...” (Slavin, 2006, p. 2). Furthermore, Vygotsky, a human development psychologist, expressed that children develop cognitively through playing (Slavin, 2006, p. 2). Playing motivates children to learn.

Teaching that integrates both aspects of learning will therefore provide an atmosphere for the youngsters to thrive in school. The application of sand drawing, a culturally based activity, will satisfy both purposes in children. They will be motivated by drawing in the sand while counting the number of dots printed. Following their observation of the physical objects, they will then convey the number of dots through prints. This method of teaching should be utilized to achieve higher level of learning. This conforms with what Piaget said about children’s learning because according to Piaget, as children grow into preadolescence, they begin to think “abstractly” (Slavin, 2006, p. 2).
History

Sand drawing follows the concept of the Euler path and the Euler circuit. This mathematical term was named in honor of Leonhard Euler, a mathematician from Konigsberg who solved the seven bridge problem in 1736. The people of his town wondered about the possibility of crossing all the bridges once and ended back up at their home, and he was able to prove that such a route was impossible (Ascher, 1991). The Euler path is represented by a graph that can trace all the different points at one sweep without retracing or interruption. The Euler circuit on the other hand, is defined as a graph that traces back in full circle to the starting point when it is completed.

Sand drawing has a rich history. It has been practiced in different parts of the world for decades with different usages. In central part of Africa in Zaire (Republic of Congo), the Bushoong and the Chokwe tribes both practice this form of art. In Marcia Ascher’s book, Ethnomathematics: A Multicultural View of Mathematical Ideas, she indicated that sand drawing is a children’s game among the Bushoong. This is a subgroup in the Kuba chieftain that is well known for its decorative work of art (Ascher, 1991). Among the Chokwe, also known as Tshokwe, sand drawing is part of a ‘storytelling tradition’ which is solely done by men (Ascher, 1991). Their figures are called sona. Young Chokwe boys learn to draw during their initiation rite. The more difficult sona is passed down from the expert story teller to their male descendant (Gerdes, 1999).

On the other side of the world, in the Pacific Ocean, Vanuatu, a group of Pacific islands also practices sand drawing. Like the Chokwe, Nitus, the drawings, are also passed down from generation to the next, and are also performed by men only. Here, however, sand drawing is neither a game nor a storytelling tools, but a “passage to the Land of the Dead” (Ascher, 1991, p. 20). The dead are expected to complete a half erased drawing guarded by a ghost. If they successfully complete it, they will be accepted, otherwise they will be eaten. Euler is still of central concern. A drawing done in an Euler circuit is known as a suon figure. Similar to the previously mentioned cultures, Vanuatuans also construct the dots before the tracing; however, they are not “considered a part of the figure” (Ascher, 1991, p. 20).

On Polowat, an island in the western region of Chuuk State (one of the four states that make up the Federated States of Micronesia) we also practice sand drawing. Little is known of the introduction of the sand drawing to Polowat. Because the Polowatans are recognized to be ‘seafaring people,’ it would be safe to assume that they brought sand drawing home from one of their voyages to the other islands (Gladwin, 1970, p. 15). Its continuous use is due to the fact that Polowat is an atoll surrounded by sandy beaches.

Similar to the Bushoong, sand drawing is one of the various challenging games played among the Polowatan children on the beach. Other games of challenges are more physical such as wrestling and cock fighting, but sand drawing is a game to test capabilities of critical performance of complex thinking.

To begin, first, the dots are printed in the sand using the index finger and the thumb.

After the dots are all printed to the challenger’s satisfaction, the tracing begins. The index finger, or a stick is used to trace around the dots. Normally a person starts from the upper left corner (because that is how it is first observed) and traces it all the way down diagonally. Then the person turns it upward. Tracing is often drawn diagonally until the end. One may choose any path to take to complete the drawing. The Euler path and Euler circuit are considered.
Part 2: Goal of Lesson Plan

This ethnomathematics-based lesson plan implements the use of sand drawings with the hope of bringing an activity into the kindergarten classroom. The lesson plan will apply the cultural richness in the teaching of mathematics. The sand drawing activity will focus on the Counting and Cardinality standard.

- Count to 100 by ones and by tens.
- Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
- Write numbers from 0 to 20.
- Understand the relationship between numbers and quantities; connect counting to cardinality.
- Count to answer “how many?”
- Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.
- Compare two numbers between 1 and 10 presented as written numerals.

Part 3: Methodology

This lesson encourages complex thinking, one of the General Learning Outcomes of the State of Hawai‘i Department of Education. It also incorporates learning of geometry, colors, and the alphabets.

To begin, the teacher needs to engage the students’ attention, and motivate them in the lesson. This can be accomplished by playing the sand drawing game. The concepts and objectives of the game need to be explained. Students will be told that in order to win, all the dots should be traced. They should also be told that retracing is not allowed, nor can they lift up their marker to continue and connect the drawing.

Considering that the students are kindergarteners, the activity will start off with one dot then gradually progress to more complex numbers of dots. The teacher will ask for a volunteer to go up to the board and draw one dot and trace around it. The rest of the students will follow along on their own paper. This strategy will also assess
students’ prior knowledge of numbers. As each student draws a number of dots, the teacher will know whether or not the student understands the quantity of the numbers orally communicated.

A variety of concepts can be integrated in this particular lesson. After tracing each group of dots, the teacher can incorporate geometry by asking questions about the shape they drew. The teacher may also solidify students’ knowledge of the different colors by asking them to color the dots. Alternatively, labeling the dots using the alphabet will strengthen student learning. Color coding or labeling the dots can also help the teacher to easily refer to a specific dot. Constructing, tracing, and counting the dots should be done repeatedly up to whatever amount the teacher decides upon. The Euler path or circuit cannot be achieved with certain sets of dots.

Elaboration and extension of the lesson needs to be planned because of the great learning diversity among the students. A way to elaborate on the lesson is to use manipulatives. Students can construct a structure to represent the dots. To extend the lesson, the teacher may introduce the word form for the numbers. Additional worksheets on this sand drawing lesson are also created to accommodate those who cannot do either the drawing or the use of manipulatives.

Part 4: Conclusion

It is proposed that a lesson based in ethnic culture and a fun activity support a thriving mathematics lesson, and create an opportune place for safe inquiry and exploration. As the students draw, they will discover more of their talents that haven’t yet surfaced. Furthermore, this lesson is thoughtfully designed with intentional teaching strategies and fosters all the mathematical domains of the Kindergarten Common Core State Standards in Mathematics. Although it is for kindergarteners, it may be used for higher grade levels such as complex computer programming courses. Sand drawings may be adjusted to accommodate any level of mathematics.

References
Sand Drawing Activity Sheet

Name: _______________________________________                Date:  ______________________

Directions: Trace around the dots, then write the amount on the blank.

Example: Dots (rectangular shape)

---

1. __________
2. __________
3. __________
4. __________
Answer Key

Sand Drawing Activity Sheet

Name: _______________________________________                Date:  ______________________

Directions: Trace around the dots, then write the amount on the blank.

Example: Dots (rectangular shape)

1 DOT

2 DOTS

3 DOTS

4 DOTS

5 DOTS
Part 1: Introduction and History

Every culture has traditions that are passed down from generation to generation. These traditions are important portals to the past. They are also what keeps the history of our culture alive and thriving. There are so many things that make Hawai‘i unique, traditions being one of them. One tradition that is making a comeback is the art of Hawaiian quilting. This tradition was once heavily guarded within ‘ohana, and it was the role of tutus, mothers, and aunties to pass down patterns and to instruct younger quilters (Arthur, 2002). Since then the art form has come out from behind closed doors, allowing people around the world to celebrate the beauty and history of Hawai‘i in such a unique and bold way.

Before the introduction of woven fabrics to the islands, Hawaiian women would spend hours beating kapa, a bark cloth made from the inner bark of the wauke (paper mulberry) plant. This bark cloth was dyed and elaborately decorated with geometric block prints and was used for bedding and festive clothing (Arthur, 2002). With the arrival of Westerners, new fabrics and sewing techniques were introduced to the Hawaiian women. In the 1820s missionary wives taught the Hawaiian women how to quilt, which soon replaced kapa making (Serrao et al., 2007). The missionary wives taught them the early American style of quilting called patchwork, where small scraps of fabric were sewn together to create patterns. To the Hawaiian women, this method seemed illogical: why would one cut a large piece of material into small strips only to sew it back together again? This inspired the Hawaiian women to incorporate kapa traditions within the designs of their quilting (Arthur, 2002).

There is an old story that, on one sunny day, a Hawaiian woman lay fabric in the grass to dry, and when she went to collect it, she noticed a leafy shadow cast onto the fabric by the overhead breadfruit tree. She quickly got her scissors and cut out the shadowy pattern. When the fabric was initially folded into fourths and then cut, the unfolded fabric displayed a perfectly radially symmetrical design. This design was then placed on a bigger piece of fabric and hand stitched to it, and thus the first Hawaiian appliqué quilt was born.

This technique quickly found its way through the islands, and before long, Hawaiian women were creating unique quilts depicting...
“Every stitch had a meaning and every part of the design had a purpose.” - Serrao, 1997

Not only were traditional Hawaiian quilts created to depict the natural beauty of the islands, but they also served as a way to keep Hawaiian culture and history alive. With the overthrow of the Hawaiian monarchy, the Hawaiian flag was lowered. Fearing that they would never see their flag again and as a sign of silent protest, Hawaiian quilters incorporated their flag along with other symbols of old Hawaiian royalty into their designs (Root, 2001). Examples of this Kuʻu Hae Aloha (“My Beloved Flag”) quilt are on display at museums across Oʻahu (Brandon, 1993). These quilts were hidden and kept out of sight in many homes. However, some brave quilters created reversible quilts with the top side being a traditional Hawaiian pattern and on the underside their proudly quilted Hawaiian flag.

On the surface it has been the evolution of an entirely unique method of quilting. Underlying, it is the embodiment of the spirit of a people, rich in creativity and sensitivity, who have shared, through this art form, not only their history and personal observations - but also their feelings and sentiments during a time in their lives, filled with extraordinary change and emotion (Root, 2001). The art of traditional Hawaiian quilting an integral part of Hawaiian history, rich with beauty, balance, and cultural pride. It is important that this unique art is not lost but instead be passed down to the generations to come.

Part 2: Goal of Lesson Plan

The goal of this lesson is to introduce students to the rich cultural history behind traditional Hawaiian quilting and find practical mathematical applications in this unique art form. Some of the common mathematical concepts that can be found in quilting are measurement, shapes, symmetry, area and perimeter, patterns, fractions, and coordinate plotting. An additional goal of this lesson is the overall growth of the student. A few General Learner Outcomes (GLOs) to focus on while doing this lesson include:

◊ **Self-Directed Learner**: the ability to be responsible for one's own learning;

◊ **Complex Thinker**: the ability to demonstrate critical thinking and problem solving;

◊ **Quality Producer**: the ability to recognize and produce quality performance and quality products (Hawaii State Department of Education, 2012).

In addition, we as teachers must hold ourselves to the highest level of our profession. A lesson incorporating local culture into mathematics can help satisfy state regulated teacher performance standards, such as:

◊ **Standard #2: Learning Differences**: The teacher uses understanding of individual differences and diverse cultures and communities to ensure inclusive learning environments that enable each learner to meet high standards;

◊ **Standard #5: Application of Content**: The teacher understands how to connect concepts and use differing perspectives to engage learners in critical thinking, creativity, and collaborative problem solving related to authentic local and global issues;

◊ **Standard #7: Planning for Instruction**: The teacher plans instruction that supports every student in meeting rigorous learning goals by drawing upon knowledge of content areas, curriculum, cross-disciplinary skills, and pedagogy, as well as knowledge of learners and the community context (Hawaii Teacher Standards Board, 2012).
Part 3: Methodology

There are many possible lessons that can incorporate Hawaiian quilting. Here are a few seeds that have the potential to grow into lessons. All of these ideas are based on the Common Core math standards for elementary grades. However, they can be adapted and developed to satisfy secondary standards as well.

SHAPES & PATTERNS:

Although very important in the younger elementary grades, these concepts could still serve as fun projects done with older students. Bring a simple Hawaiian quilt into class or have a large picture of one available for students as a reference. Using the simple design of the appliqué, have students use tangrams and/or pattern blocks to recreate part of the design. This gives students great hands-on practice with identifying shapes and exploring shape orientation. It also allows them to see patterns within the quilts. Using these differently-shaped blocks is one way to let children’s imaginations run wild while teaching important concepts.

MEASUREMENT:

Bring a quilt or two into the classroom for students to see and experience firsthand. Have younger students see and explore the measurable attributes of the quilt (e.g., length and width). Have older students physically measure a quilt or make one of their own using standard and nonstandard measurements. This could also be a way to introduce and practice using area and perimeter. Again, have students work with real quilts to find the amount of fabric needed to make a border of the quilt or how large of an area the quilt will cover. Giving them these hands-on opportunities will solidify what they are learning. The possibilities for teaching measurement through quilting are endless!

FRACTIONS:

To create a traditional Hawaiian quilt, the fabric must be folded in half (½), and then again (¼) and again (⅛). There are so many different fractions that can be seen and used in quilting. Create Hawaiian quilt fraction puzzle cards for students to play with. To make these, take pictures of Hawaiian quilts, or create your own, and cut them into different fractions. The students can then use the cards to practice their fractions by matching the quilt designs. Another idea is to give students pieces of paper to fold and from which to cut out their own designs. Then have the students color and label the appropriate fraction. These can then be hung around the classroom as beautiful decorations and as visual reminders of what the students learned.

SYMMETRY:

Symmetry is often the first most prominent mathematical concept when looking at traditional Hawaiian quilts and rightfully so. These quilts were designed to be symmetrical and balanced, therefore giving us a perfect tool to teach symmetry! There are often many lines of symmetry within a single traditional quilt. Show your students pictures of many different quilts (see “Teacher References”), or better yet, take them to the Mission Houses Museum to show them one of the largest collection of Hawaiian quilts. During your visit ask the students to find the number of lines of symmetry in each of the quilts. The paper folding activity is also a great tool for teaching symmetry, and it allows the students to design their own traditional quilt pattern. For a more high-tech version of this, visit “Math is Fun! Symmetry Artist 2” online (within “Teacher References” under “Websites”). Students are able to adjust different variables to create an array of patterns. Using this interactive program, have students create and share their own quilt patterns.

TEACHER REFERENCES:

BOOKS - The following offer a wealth of information and have beautiful, large, color pictures.

◊ Contemporary Hawaiian Quilting by Linda Arthur;
◊ Hawaiian Quilt Masterpieces by Robert Shaw;
◊ Hawaiian Quilts, Tradition and Transition by Reiko Brandon and Loretta Woodard;
◊ The Hawaiian Quilt by Reiko M. Brandon;
◊ The Hawaiian Quilts, the Tradition Continues by Poakalani Serrao.

STORY BOOK - Luka’s Quilt by Georgia Guback is a fun and beautifully illustrated children’s book -- a great way to lead into the lesson.

WEBSITES - These sites provide history, instruction, pictures, and online teaching tools:

◊ Math Is Fun! Symmetry Artist 2: http://www.mathsisfun.com/geometry/symmetry-artist2.html; and
Part 4: Conclusion

This is only a brief introduction into the beautiful art form of traditional Hawaiian quilting and some of the ways we can use it to enrich our students’ learning experience. Hawai‘i is host to rich and beautiful culture, and the purpose of lessons like this is to bring our students out of the classroom and into a world of extraordinary learning opportunities. Let these small lesson “seeds” take root in your class and they will grow into beautiful experiences for your students. A quilter once said, “Every stitch had a meaning and every part of the design had a purpose” (Serrao, 1997, p. 112). May you keep these words with you as you stitch and weave local culture into your teachings and classroom, giving every lesson a deeper more meaningful purpose. Aloha.

MUSEUMS - Here are a few museums within the state that have traditional Hawaiian quilts on display:

◊ Island of O‘ahu: Mission Houses Museum, Bishop Museum, Honolulu Museum of Art;
◊ Island of Kaua‘i: Kaua‘i Museum; and
◊ Island of Hawai‘i: Mauna Kea Beach Hotel.

Original quilt design by Jessica Evans

References

Three

Mathematics and Native Hawaiian Plants

Included in this chapter:
◊ Geometric Shapes in Nature - Worksheet 1
◊ Botanical Garden Field Trip Activity - Worksheet 2

Written by Donna G. Soriano

Part 1: Introduction and History

Educators are often challenged to find ways of engaging students in learning. Students can get bored very easily, and younger students may not absorb much information from just a lecture. Teachers want their students to enjoy the subject they are learning, so that students will want to learn and will pay attention. One way to engage them is to take the learning outside of the classroom -- especially for those students who often stare out of their classroom windows, wishing they were playing outdoors. How might teachers take mathematics lessons beyond four cement walls, so that students might be more joyful and engaged?

Many opportunities exist that can make concepts real and relevant to students by placing them in more realistic environments. When students are exposed to these types of learning contexts, their minds are freer to explore and be creative, regardless of the subject they are learning. Certain concepts can be particularly challenging for some students. Taking students outside may prevent frustration by making mathematical concepts relevant, since subjects are often easier to grasp when set in context. In an outdoor classroom, imaginations can run wild, so prepare to be amazed by the wonderful ideas students come up with as a result of being excited about learning.
Many mathematical concepts are present in nature and culture. In Hawai‘i we are fortunate to be surrounded by vibrant tropical plants, many of which have long histories of use by Native Hawaiians. As masters at of horticulture, Native Hawaiians made good use of nature for a variety of purposes. They used kukui nuts to make candles, hala for baskets and cordage, naio, ki, pili, and lama in the construction of their houses, wauke and ʻakala to make kapa for clothing, and koa to make canoes and boards (Krauss, 1974). They also used plants to make musical instruments, in games and sports, for medicinal purposes, and for weaponry and caskets. Taking students outside and into nature’s classroom enables them to learn about Hawaiian culture, mathematics, and history.

Part 2: Goal of Lesson Plan

Among the goals of this lesson is to have students become more aware of their natural surroundings. Some Hawaiian plants are especially unique because they cannot be found anywhere else on the planet. By incorporating mathematics, nature, and culture, teachers can help their students will develop a sense of appreciation for the environment in which they live.

As the students participate in the activities, they will become familiar with the common names and histories of some traditional Hawaiian plants. Students will ideally be able to identify some of these plants whenever they see them outside in nature. They will also learn how to observe number patterns, such as the Fibonacci sequence in a plant’s flower petal count, leaf count, and seed head counts. As they their tabulate data, they practice using tables and recording observations. Students will also observe symmetrical and asymmetrical patterns in Hawaiian plants as well as identifying geometric shapes in plants’ physiological structures.
Some of the standards and benchmarks addressed by this lesson include:

◊ Categorize and justify a number as being odd or even.

◊ Create and describe growing numerical and spatial patterns and generalize a rule for the pattern.

◊ Cultural Anthropology: Systems, Dynamics, and Inquiry – Understand culture as a system of beliefs, knowledge, and practices shared by a group and understand how cultural systems change over time.

Activities in this lesson address Common Core State Standards for Grade 4 Geometry (4.G.3.): Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry (Common Core State Standards Initiative, 2012).

Part 3: Methodology

In the first activity, the teacher engages the students by sharing moʻolelo (legends) with them that are associated with certain plants as well as these plants’ traditional uses. The teacher will then have each student choose a specific plant on which to do a short mini-research project that he or she will share with the rest of the class. This activity will help students familiarize themselves with the plant names, history, and uses.

The second activity will allow students to explore the features of traditional Hawaiian plants by identifying geometric shapes observable within them. The students are given a worksheet with pictures of Hawaiian plants and are tasked to write down all of the geometric shapes they can identify within the picture. They will also quantify how many of each geometric shape they see within the flower, the fruit, the stems, and the leaves of the plant. This activity can also serve as an art project, where students use geometric-shaped sponges (and paint) to create their own plant print. The students could also compose a moʻolelo about one of the plants.

The next activity involves students observing number patterns in their natural surroundings. The students will participate in a field study at one of Hawaiʻi’s many botanical gardens, which on some of the familiar gardens on Oʻahu include: Hoʻomaluhia Botanical Garden, Harold Lyon’s Arboretum, Foster Botanical Garden, and Hawaiʻi’s Plantation Garden. Students can work in groups of three or four to answer questions on their activity sheet. They will learn to identify the native (and non-native) Hawaiian plants and learn about quantifying and tabulating data by counting the number of leaves, number of flower petals, or seed heads on certain plants. See if they can find a pattern in the numbers they see in nature or relationship to the Fibonacci sequence: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, etc. For example, the number of flower petals on a Hibiscus is 5, a Fibonacci number. Have a discussion about why Fibonacci numbers are commonly found in nature and depicted in art.

Another activity could have students identifying symmetry in traditional Hawaiian plants. The teacher can choose several Hawaiian plants from which to generate an activity worksheet, where students have to draw each plant’s line of symmetry. The teacher could also include some Hawaiian plants that are not as obviously symmetrical, such as the noni plant. An additional challenge would be to have the students draw the reflected half of such plants as if they were symmetrical.
Part 4: Conclusion

It is very important for students in Hawai’i to be aware of their natural environment. Some Hawaiian plants deserve special attention because of their uniqueness to Hawai’i and because they are constantly being threatened by invasive plant and animal species and human activities. Hopefully this lesson will encourage your students to want to learn more about Hawaiian plants: how they developed unique features, how they were used, and how they fit into our island ecologies. If students are knowledgeable about plants, then they may be more inclined to help protect those that are endangered and, perhaps, even help them become reinstated and preserved in nature. That way, these plants will be around for future generations living in Hawai’i to also learn about and enjoy.

References


Worksheet 1
Geometric Shapes in Nature

Identify the geometric shapes you see in the various Hawaiian plants. How many of each geometric shape can you find in each of the plants?

Hexagon  Circle  Square  Rectangle  Triangle  Pentagon  Diamond  Star

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<tr>
<th>POHUEHUE</th>
<th>![POHUEHUE Image]</th>
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<td>![ALOALO Image]</td>
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<td>Date</td>
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<th>‘ŪLEI</th>
<th>![Image of 'ŪLEI]</th>
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<tr>
<td>PUA MELIA</td>
<td>![Image of PUA MELIA]</td>
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<td>KOKI'O</td>
<td>![Image of KOKI'O]</td>
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</table>
Worksheet 2
Botanical Garden Field Trip Activity: Number Patterns in Nature

<table>
<thead>
<tr>
<th>Plant Name</th>
<th>Image of Plant</th>
<th>Flower</th>
<th>Seed Heads</th>
<th>Leaves</th>
<th>Leaf Veins</th>
<th>Fruits</th>
<th>Misc.</th>
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</thead>
<tbody>
<tr>
<td>PALAPALAI Native Hawaiian Fern</td>
<td>Native Hawaiian Fern</td>
<td>Microlepia Strigosa</td>
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<tr>
<td>HALA Screwpines Tree</td>
<td>Screwpines Tree</td>
<td>Padanus Tectorius</td>
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<tr>
<td>KOKI‘O Red Hibiscus</td>
<td>Red Hibiscus</td>
<td>Hibiscus Koki‘o</td>
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<tr>
<td>‘ŌHI‘A LEHUA Metrosideros Polymorpha</td>
<td>‘ŌHI‘A LEHUA</td>
<td>Metrosideros Polymorpha</td>
<td></td>
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<tr>
<td>LOULU Native Hawaiian Palm Pritchardia Hardyi</td>
<td>LOULU</td>
<td>Native Hawaiian Palm</td>
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<td>Plant Name</td>
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<tr>
<td>'AWAPUHI 'ULA'ULA</td>
<td>Hawaiian Red Ginger</td>
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<td>Alpina Purpurata</td>
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<tr>
<td>ALOALO</td>
<td>Native Yellow Hibiscus</td>
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<td>'ULEI</td>
<td>Hawaiian Rose</td>
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<td>Osteomeles Anthylidifolia</td>
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<td>LAUA'E</td>
<td>Maile-Scented Fern</td>
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<td>Cordyline Fruticosa</td>
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<tr>
<td>Plant Name</td>
<td>Image of Plant</td>
<td>Flower</td>
<td>Seed</td>
<td>Heads</td>
<td>Leaves</td>
<td>Leaf</td>
<td>Fruits</td>
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<tr>
<td>HELICONIA Lobster Claw</td>
<td>Heliconia Caribaea</td>
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<td></td>
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<tr>
<td>LO'I KALO Taro</td>
<td>Colocasia Esculenta</td>
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<tr>
<td>NONI Beach Mulberry</td>
<td>Morinda Citrifolia</td>
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Section Two: Middle/High Level
Many students have a great fear and anxiety of mathematics. In high school many students ask the teacher, “When are we ever going to use this?” These students are unaware that mathematics is everywhere and in our everyday lives. Simple and unnoticed daily actions can easily be described using mathematics; for example, if you don’t do your homework (x-value), then you will receive a poor grade (y-value). Ubiratan D’Ambrosio coined a term he called “Ethnomathematics”. This term may sound intimidating, but it is basically the study and relationships between mathematics and culture (D’Ambrosio, 2001).

Dr. Linda Furuto, mathematics professor at the University of Hawai’i West O’ahu, said, “Mathematics is not solely contained within the four sterile walls of the classroom but it is everywhere we look, everywhere we touch from the oceans to the mountains to the heavens”. This statement by Dr. Linda Furuto is true in many ways, students who have mathematics anxiety don’t even realize they are doing mathematics at times. Having the students engage in real-world situations, that are relevant to their culture and history will solidify the connection of mathematics to their life.

By incorporating mathematics and culture, this lesson plan will utilize Hawai‘i’s unique environment and native plant species. The purpose of this Ethnomathematics lesson plan is to engage students with our rich history and culture so they understand the basic concepts of geometric equations, and linear functions. The lesson will also focus on how invasive plant seeds may disperse and affect Hawai‘i’s native plants in the event of strong winds.

Hawai‘i has a very diverse cultural and natural aspect, which attracts many tourists who seek to relax in paradise and a warm year-round tropical climate for honeymoons, marriage, or leisure. Hawai‘i also lures many worldwide scientist and researchers because it houses the second most active volcano in the world, Kilauea, on the Big Island. Although Hawai‘i’s population is exponentially growing, we must remember that Hawai‘i was not always as urbanized as it is now.

About 3,000 years ago, the Polynesians began to explore the numerous tiny islands in the grand Pacific Ocean, and on their
When they arrived, an abundant amount of marine food was available, however minimal edible plants/vegetation were available. Luckily, they brought along with them breadfruit, taro, bananas, and much more.

Did you know Hawai‘i is the most isolated landmass in the world? The closest landmass is California, which is about 2,390 miles away. Due to the isolation, Charles Darwin’s theory of natural selection supports why Hawai‘i has many unique plants and animals. Over the century, there have been many accidental or introduced alien plant species that place a threat against Hawai‘i’s native plants. A great example is Brachiaria mutica, also known as California Grass; this species of grass grows as high as 2 meters, which would easily outgrow many shrubs and trees (Smith, 1998).

Preserving Hawai‘i’s natural environment is very important because our plants and animals are one of a kind. However, if winds can have an effect on the dispersal of invasive plant seeds, volunteers will need to work hard to save the native plants. We must preserve what we have now and save our endangered plant and animal species before it become a larger problem.

Part 2: Goal of Lesson Plan

The goal of this Ethnomathematics lesson plan is to provide students with mathematical knowledge and experience, to widen their perspective of how they can apply math to everyday life and ongoing trends. Students will also strengthen their skills in geometric equations, and linear functions.

This lesson plan satisfies multiple sections of the Common Core Mathematics Standards, for the seventh grade.

Grade 7: Common Core Mathematics Standards

1. 7.EE.1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

2. 7.EE.2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that “increase by 5%” is the same as “multiply by 1.05.”

3. 7.G.4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Part 3: Methodology

BASIC MATH SKILLS COVERED:

- Linear Functions
- Radius/Area of a Circle
- X/Y Table – Identifying Patterns
- Graphing

MATERIALS/TOOLS:

1. Worksheet
2. Paper
3. Pencil/Eraser
4. Calculator (optional)

BACKGROUND OF THE PROBLEM/ SPECIFIC CONCEPTS:

The ‘ Akoko is a native and endangered plant that was discovered near the steep cliffs of the Nu‘uanu Pali, on the island of O‘ahu. A widely spread invasive plant found in Hawai‘i is the California Grass that is known to cover many forests and mountainsides. Hawai‘i has many invasive plants that can have damaging effects on our native plant species. With the use of a variety of different mathematical equations we can determine where to expect invasive plant seeds. This will give us a better idea where an influx of baby invasive plants may sprout.

DETAILED DIRECTIONS:

Step 1: Review problems with your students on linear functions, area of a circle, x/y tables, and graphing. Then go over the history of this lesson plan.

Step 2: Create a linear equation with the given information below.

*See conclusion for alternative scientific activity

Assume 1 mile per hour (mph) winds can spread seeds up to 2 feet. Create a linear equation (ex. y = mx + b) to use with the following question. State what your variables are.
A group of student volunteers are planning a service learning project that is done monthly. Over the past month the wind speeds in Nu‘uanu were equal to or less than 20 mph. If the ‘Akoko is 50 feet away from the invasive plant (California Grass), should there be a concern of California Grass seeds near the base of the ‘Akoko?

**Student Answer(s):**

\[ y = 2x + 0 \text{ or } y = 2x \]
Where: \( y = \) Radius in feet (ft)
\( x = \) Wind speed in miles per hour (mph)

Step 3: Students will find the radius with the equation they created; they will then calculate the area of the circle to answer the question in the problem below.

**CALCULATING THE RADIUS**

\[ Y = 2x + 0 \]
\[ Y = 2(20) + 0 \]
\[ Y = 40 + 0 \]
\[ Y = 40 \text{ ft} \]

Calculating the Area

\[ \text{Area} = \pi \times r^2 \]
\[ \text{Area} = \pi \times 40^2 \]
\[ \text{Area} \approx 5026.55 \text{ ft}^2 \]

**Student Answer(s):**

With 20 mph winds there shouldn’t be concern about the invasive plant seeds being blown to the base of the ‘Akoko.

**Student Reflection:**

1. Why is this information (radius/area) helpful for the prospective volunteer group?

Possible response (Responses will vary):

Finding the area around the invasive plants will allow the student group to determine the general area that will need to be cleared. Knowing this information is important because the volunteer group can use the area size to estimate the amount of workers they will need to clear the land.

**Step 5 Exercises**

**X/Y TABLE**

Instructions: Based on the question above, about what wind speed would the invasive plant seed’s drift to the base of the ‘Akoko? Use the table to plot points and graph. Circle your answer on the table.

To answer this question the student should create a X/Y table. Example below:

<table>
<thead>
<tr>
<th>X (Wind Speed)</th>
<th>Y = 2x</th>
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</thead>
<tbody>
<tr>
<td>20 mph</td>
<td>40 Feet</td>
</tr>
<tr>
<td>21 mph</td>
<td>42 Feet</td>
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<tr>
<td>22 mph</td>
<td>44 Feet</td>
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<tr>
<td>23 mph</td>
<td>46 Feet</td>
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<tr>
<td>24 mph</td>
<td>48 Feet</td>
</tr>
<tr>
<td>25 mph</td>
<td>50 Feet</td>
</tr>
</tbody>
</table>

The student should be able to distinguish the pattern.
Solution:

Based on the X/Y table we can conclude that 25 mph winds would show signs of the invasive plant seeds at the base of the 'Akoko.

GRAPHING

Question for Students:

1. What quadrants should you focus on Quadrant I, II, III, IV? Explain.

Responses will vary: Quadrant I. It is impossible to have a negative speed of wind, such as -1 mph.

Instructions: Using the points the students solved above, have them graph it. This graph will be used to reinforce their written response for the question above.

Example of the graph

Step 6: Concluding response questions

1. What other real-world applications can you use linear functions and geometric equations on?

2. Critical Thinking: In your own words describe what you think math is and why it is important?

Part 4: Conclusion

At the conclusion of this lesson plan students should have a stronger understanding about how they can relate their personal history, culture, and everyday world situations to linear functions, radius/area of a circle, x/y tables, and graphs. If you would like to take this lesson plan further great field trips/activities are listed below.

◊ Many organizations around Hawai‘i arrange clean ups to restore and clear land for Native Hawaiian Plants. Not only will your students be actively engaged in math, they will be servicing their community.

◊ Alternative for “Step 2: Create a linear equation with the given information below”.

Instead of having the students use the given information to create a linear function, conduct a science lab. Please note that answers will be different if this option is taken.

Materials: Shredds of paper, tape measure, anemometer.

Steps:

1. Using the anemometer measure the wind speed in miles per hour (mph). Be sure the person measuring the wind speed does not move (they will be a reference point).

2. Throw the shreds of paper in the direction of the blowing wind.

3. Once the shreds of paper settle, measure the distance in feet (from the wind measurer to the closest paper shred).

4. Calculate how far the paper traveled for every mph of wind. Example: If the wind is measured at 15 mph and the nearest shred of paper is 13 feet, the paper traveled about 13 / 15 = .87 feet for every 1 mph of wind.

5. Use your information to create the linear function on how far the invasive seeds will travel, assuming that the shreds of paper resembled the seeds of the invasive plant.

** If an anemometer is unavailable you may use this resource: http://www.energyquest.ca.gov/projects/anemometer.html
The ‘Akoko is a native and endangered plant that was discovered near the steep cliffs of the Nuʻuanu Pali, on the island of Oʻahu. A widely spread invasive plant found in Hawaiʻi is the California Grass that is known to cover many forests and mountainsides. Hawaiʻi has many invasive plants that can cause damaging effects on our native plant species. Invasive plants can cause suffocations due to their overgrowth. With the use of a variety of different mathematical equations we can determine where to expect invasive plant seeds. This will give us a better idea where an influx of baby invasive plants may sprout.

**Problem:**
A group of student volunteers are planning a service learning project that is done monthly. Over the past month the wind speeds in Nuʻuanu were equal to or less than 20 mph. If the ‘Akoko is 50 feet away from the invasive plant (California Grass), should there be a concern of California Grass seeds near the base of the ‘Akoko?

1) Use the given information below to create a linear equation which solves for y, when \( y = \text{radius} \). State your variables.
Assume 1 mile per hour (mph) winds can spread seeds up to 2 feet. Create a linear equation (ex. \( y = mx + b \)) to use with the following question.

2) Using your equation that you created. Find the radius for the circle below.

3) Using the answer that you found in question number 2. Find the area of the circle.

4) If the ‘Akoko is 50 feet away from the invasive plant (California Grass), should there be a concern of California Grass seeds near the base of the ‘Akoko?
5) Why is this information (Radius/Area) helpful for the prospective volunteer group?

6) What wind speed would the invasive weed seed’s have drifted to the base of the ‘Akoko? Use the table to plot points, and graph. Circle the wind speed on your table.

<table>
<thead>
<tr>
<th>X (Wind Speed)</th>
<th>Y (Radius)</th>
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7) Using graph paper, graph the points that you calculated above. What quadrant should you focus on? (Circle answer)

   Quadrant I       Quadrant II       Quadrant III       Quadrant IV       None
   Why?
Part 1: Introduction and History

Taro, or kalo, is one of the oldest cultivated crops in the world. Mentioned in Chinese writings as early as 100 B.C., kalo was believed to have originated in India or Southern Asia (Krauss, 1993). Kalo reached Egypt and the Mediterranean parts of Europe about 2,000 years ago, where it subsequently spread throughout the Pacific Area (Nonnecke, 1989). According to Kirch (1985), the Polynesians arrived in Hawai‘i around 1100 A.D. bringing with them an array of foods, including kalo. The kanaka maoli, the indigenous Hawaiian people, consumed kalo on a daily basis where each person was thought to eat approximately 5-15 lbs. of poi (Ebel & Mak, 1974). Beyond a dietary staple, kalo played a much greater role extending into the spiritual center of Hawaiian culture.

According to Hawaiian mythology, Wākea and Ho‘ohokukalani, the first couple in the creation chant, gave birth to Hāloa-naka, a stillborn child (Krauss, 1993). Wākea buried his lifeless child near the house. Shortly thereafter, the kalo plant sprouted out of his body. The first couple’s second son was named Hāloa after their first-born. Through Hāloa, the human race and Hawaiian people descended. Hāloa was to respect and look after (mālama) his older brother or the kalo for all eternity. Reciprocating, the first born son would sustain and nourish his younger brother and his posterity. Having descended from the second son, many native Hawaiians viewed kalo as being genealogically superior to themselves (Krauss, 1993). The value of kalo was elevated and perpetuated through this creation myth as evidenced in their language, traditions, rituals, customs, and technology.

For example, the Hawaiian terminology for family, ʻōhana, is derived from a part of the kalo called ʻōha. ʻŌha is referred to as the corm or the underground stem of the plant (Krauss, 1993). Other parts of the kalo plant were used in the process of dying kapa cloth and lauhala. The leaf-stem juice and the mud from the taro patch were used to make red and black dye, respectively. In addition, selected varieties of kalo were used as
offerings to the gods. The 300 different varieties of taro also revealed the social stratification system of Hawai‘i. Producing a pinkish and purplish poi, the royal taros were exclusively reserved for the high chiefs and royalty whereas the gray and white taros were allotted for commoners. Other varieties of taro were used for medicinal purposes in healing and as offerings to the gods. Over the several centuries following the initial arrival of Polynesians, there began a progressive expansion of taro production in accordance with the expanding Hawaiian population. Around 1650 A.D., the Hawaiian population expanded over 400,000 people and was sustained through an impressive level of engineering innovations (Cho, Yamakawa, & Hollyer, 2007).

Ancient Hawai‘i was widely recognized as the zenith in technological sophistication with respect to the cultivation of taro around the world. Donald D. Kiolani (1982) Mitchell, an author of Hawaiian history and culture, referred to the ancient Hawaiians as ‘the most sophisticated horticulturalists’ (as cited in Kanahele, 1986, p. 301). With the two types of taro cultivation - dry land and wetland, the native Hawaiians favored the latter as it was 10 to 15 times more productive than dry unirrigated taro gardens (Kelly, 1989). The complex terraced pond fields (lo‘i) and intricate irrigation system (‘auwai) proved to be an efficient means of producing wetland taro and an undeniable feat of engineering, ingenuity, and skill (Cho et al., 2007). As an act of reverence to the kalo plant, only men could plant, harvest, and mash poi; however, the women would also mash poi when the men were not present (Krauss, 1993).

The stone-faced terraced ponds were irrigated by water from streams in the valleys or springs below the surface, which allowed extensive areas of the valleys to be cultivated. Mastering the rate or flow of water dispersed throughout the different terraces, the native Hawaiians were able to find the angles of elevation that would prevent the taro corms from rotting. Without proper circulation, the water would stagnate and overheat causing rot, but at the same time an overly intense flow of water would erode the ditches and terraces (Kelly, 1989). As recent as 2009, scientific research has found that early Hawaiian agriculture may have been more complex and extensive than previously thought (Nature). Utilizing Geographical Information Systems (GIS) mapping and previous archaeological evidence, scientists have found empirical proof of how extensive the taro and farming lands were in ancient Hawai‘i, which would account for the ancient Hawaiians ability to support a population of approximately 1 million people (Kirch & Rallu, 2007).

After the arrival of Captain Cook, taro cultivation began to decline because of a variety of subsequent causes and events: decrease in Hawaiian population, loss of agricultural knowledge, alternative foods from other countries, the Great Mahele of 1848, shift to cultivating rice, and change in legal rights that diverted water elsewhere (Cho et al., 2007). Challenges to cultivating taro in modern times was further complicated with the biotech industry, which seeks to genetically modify taro in order to create a more disease-resistant version. The spiritual connection and cultural beliefs had many demonstrators, activists, and protestors supporting legislation that places a moratorium on the genetic modification of their revered kalo plant (Niesse, 2007).

Part II: Goals of Lesson Plan

1. **Objective:** Identify the cultural importance of kalo in Hawaiian history.

2. **Objective:** Explain the Creation Myth and tie the concepts of ‘Ohana and Hāloa back to kalo.

3. **Objective:** Explain how language, traditions, rituals, customs, and technology are elements of culture.

4. **Objective:** Bring awareness to the contentious issue of kalo cultivation and experimentation utilizing GMO technology.

5. **Objective:** Plan a field study to one of the taro farms or botanical gardens.

6. **Objective:** Meet Common Core State Standards for Mathematics in Lesson Plans.
**COMMON CORE STANDARDS:**

**8.F Define, evaluate, and compare functions.**

1. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

2. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two \((x, y)\) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

3. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

**G-MG Apply geometric concepts in modeling situations**

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

2. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

**F-LE.1. Construct and compare linear, quadratic, and exponential models and solve problems**

1. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

2. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

---

**Part III: Methodology**

After providing background information and history on kalo, the instructor should define increasing, decreasing, and constant functions prior to distributing Worksheet #1:

A function increases on an interval of its domain if its graph rises from left to right on the interval. It decreases on an interval of its domain if its graph falls from left to right on the interval. It is constant on an interval of its domain if its graph is horizontal on the interval (Lial, Hornsby, & Schneider, 2009).

- **Increasing**: function \(f\) increases over the interval if, whenever \(x_1 < x_2\), then \(f(x_1) < f(x_2)\)

- **Decreasing**: function \(f\) decreases over the interval if, whenever \(x_1 < x_2\), \(f(x_1) > f(x_2)\)

- **Constant**: function \(f\) is constant on the interval if, for every \(x_1 < x_2\), then \(f(x_1) = f(x_2)\).

The instructor should give a brief recapitulation of the history of kalo as it pertains to Worksheet #1. Worksheet #2 should be distributed.

In Worksheet #2, the instructor may choose to review slope, circular trigonometric functions, proper interval notation, and geometric area formulas prior to distribution:

**Slope**- Geometrically speaking, the slope of a line is a numerical measure of the steepness and can be interpreted as rise over run (Lial, Hornsby, & Schneider, 2009).

\[ m = \frac{\text{rise}}{\text{run}} \]

**Trigonometric Functions**

**Law of Sines** - with any triangle, the ratio of a side length to the sine of its opposite angle is the same for all three sides of a triangle.

\[ \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \]

* In order to use the Law of Sines, there must be at least one angle and the side opposite that angle.

**Proper Interval Notation** - When using interval notation, there are specific ways to express the different types of intervals. An open interval is indicated through parentheses. For example, the interval \((-2, 8)\) is an open interval with the end points, -2 and 8.
which are NOT part of the interval. The closed interval is indicated using square brackets. The interval \([-2, 8]\) is a closed interval with the end points, -2 and 8, which ARE part of the interval (Lial, Hornsby, & Schneider, 2009).

Geometric Area Formulas - The area of a rectangle is defined as length times width. The sum of all angles in a triangle are 180 degrees.

After completion of the worksheets, the instructor can also organize a field study to one of the kalo farms or botanical gardens on O‘ahu (Kupunakalo):

1. Ka Papa Lo‘i O Kānewai Native Hawaiian Cultural Gardens, UH Mānoa: 2540 Maile Way, Spalding 454, Honolulu, HI 96822
2. Lyon Arboretum in Mānoa Valley: 3680 Mānoa Road, Honolulu, HI 96822
3. Waimea Valley Botanical Collection: 59-864 Kamehameha Hwy, Haleʻiwa, HI 96712
4. Waimānalo Research Station, College of Tropical Agriculture and Human Resources: 41-698 Ahiki Street, Waimānalo, HI 96795
5. Waianae Ka‘ala Farms, Inc.: PO Box 630 Waianae, HI 96792
6. Hui Kū Maoli ola, O‘ahu - Haʻikū Area: 46-403 Haʻikū Road Kāneʻohe, Hawai‘i 96744

* The Kupunakalo.com website features potential locations for those living on neighbor islands.

Part IV: Conclusion

The lesson plan should conclude with a group reflection that will engage students in discussion. The instructor can ask questions that highlight the significance of the kalo plant in Hawaiian history and culture. Encouraging students to share what they learned during the lesson, instructors can also elicit feelings or views on genetic modification of the kalo plant. Lastly, instructors should review math concepts and reiterate the application and interwoven nature of math to history and culture.

Reference


Based on what you learned in the lecture, what historical factors could account for the increase in taro production from the intervals of 1100 to 1650 years (x axis values).

____________________________________________________________________

____________________________________________________________________

From 1650 to 1800, the taro production and native Hawaiian population remained constant. Calculate the equation for the interval where the taro production remains constant.

____________________________________________________________________

____________________________________________________________________

Looking at the graph, the interval 1800 to 2012 (x axis) indicates a change in the graph. Name 3 events that occurred after the early 1800s that could explain this decrease.

____________________________________________________________________

____________________________________________________________________
The height of the mountain from top to bottom is 160 feet. Each terrace is 40 feet in height. According to Sheng (1989), a suitable site for terracing would have an angle measurement between 7-25 degrees. a) Find the interval measurement of x. b) Find the interval of the slope of line L.

With a 9 degree angle measurement, what would the hypotenuse or value of x be?

What would the slope be of line P?

If we adjust the angle measurement to 15 degrees, what would the hypotenuse or value of y be?

What would the slope be of line Q?

Which line is steeper? Why? (Hint: compare the slopes)
For every 43,560 square feet (1 acre), 25 people can be fed. If the four terraces are able to feed 100 people total, what is the length of a, b, c, d.
Part 1: Introduction and History

The history of Hawaiian music is both significant and relevant to students and educators. One particular musical instrument that is renowned in Hawaiian music is the ukulele. In 1879, the ukulele was first introduced with the inception of Portuguese immigrants from the island of Madeira (King John). Hawaiians were particularly impressed by the Portuguese instrument “ukulele”. Ukulele got its name from Queen Lili‘uokalani by combining the two Hawaiian words “uku” (gift or reward) and “lele” (to come) (King & Tranquada, 2003). Before contact with the western civilization, Hawaiians celebrated nature, their gods and love of life through the expression of chants and hula. Hawaiian chants come in two basic styles “Mele oli” are chants without musical instruments and “Mele hula” are songs accompanied with dance and musical instruments (James, 2005).

In the late 19th century Hawaiians developed a steel guitar which is positioned horizontally and strings are plucked with one hand, while the other hand changes the pitch of one or more strings with the use of a bar or a slide (Greenberg, 1992). This instrument is called the ki hoʻālu, or Hawaiian slack key guitar, and is truly one of the great acoustic guitar traditions in the world. Ki hoʻālu means “to loosen the key”, which is a solo finger picked style. In this tradition, the strings (“keys”) are “slacked” to produce many different sounds (Beamer, 2012).
Although, modern Hawaiian music is a combination of ancient music and it has modern influences. It is not uncommon to hear a new Hawaiian song with components of country music (Instant Hawai‘i, 2008). The lyrics usually have to do with the places of Hawai‘i’s love, nature, and culture. Thus, Hawai‘i is so diverse; this diversity has made its way into the beauty of music.

The knowledge of Hawaiian music goes throughout the classrooms and it incorporates ethnomathematics. Building methods and strategies for students to play the ukulele will enhance the understanding of math. Thus, the link between tradition and music will be exhibited through this intercultural exercise in mathematics.

Part 2: Goal of Lesson Plan

Learning the ukulele’s strings and tunings will focus on the cycle of ethnomathematics and to expand its ideas for future teachers. The goal is to teach seventh graders the relationship between ukulele’s four strings (A, E, C, G) and its frequency (Hz). A 26” Tenor ukulele will be played with three different unique tones (G4-C4-E4-A4, A4 D4 F#4 B4, & D4-G3-B3-E4) (Hurd, Costello, & Beloff, 2012). According to the Common Core State Standards for Mathematics (CCSS), grade seven will identify ratios and proportional relationships (7.RP.1, 7.RP.2, 7.RP.3) such as identifying diagraphs, testing for equivalent ratios in a table, graphing points, and relationships between quantities (Common Core State Standards Initiative, 2012).

Furthermore, this activity corresponds to Hawai‘i Content and Performance Standards III Benchmark MA.7.10.1-3: Patterns, Functions, and Algebra (i.e., tables, graphs, graphing technology, equations of linear functions, and linear relationships); and Benchmark MA.7.11.1: Data Analysis, Statistics, and Probability (i.e., pose questions, collect data, select the appropriate representation graph and display data) (Hawai‘i Standards Database, 2011). Students will learn the basic history of the Ukulele and the importance of the tones. Also, students will identify each string, analyze the three different tuning (C, G, D), record each chord (Audacity), measure the chord’s frequency through Audacity and draw a line graph to compare the four string’s frequency. Plus, visualizing the Ukulele sound waves through audacity will engage the students to recognize and distinguish the patterns of mathematics.

Part 3: Methodology

Basic Data

To initiate the thought process, the teacher will ask What does the word ukulele mean? Is the ukulele originally from Hawai‘i, if not where? What are the differences between “Mele Oli” and “Mele Hula”? After the discussion of ukulele’s history, the students will engage in a fun activity and answer each question as a class. The following table is an example how the students will record and write down data.

Audacity Data

<table>
<thead>
<tr>
<th>String</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>First String</td>
<td>A (440 Hz)</td>
</tr>
<tr>
<td>Second String</td>
<td>E (329 Hz)</td>
</tr>
<tr>
<td>Third String</td>
<td>C (264 Hz)</td>
</tr>
<tr>
<td>Fourth String</td>
<td>G (196 Hz)</td>
</tr>
</tbody>
</table>
To begin, the students will pair up with a partner and they will identify each chord on the Ukulele's strings (shown below).

1. The first student will play one string at a time while the other student is recording the sound wave through Audacity and switch turns. For instance, the following example is a recording of chord C (Third String).

2. Once the students stop recording, they will analyze the frequency by using plot spectrum to make it readable and easier to understand (see below).

   a. Go to “Analyze” menu then select “Plot Spectrum”
   b. A new window will open (Frequency Analysis) and set the “Algorithm” to Standard Autocorrelation at the bottom left (see #1).
   c. Place the mouse cursor on the graph to the first peak or the second peak (see #2) Determine the chord’s name and its frequency at the bottom (see #3).

Graphing

3. Each student will have the opportunity to have their own data table and start graphing each point. To have an accurate graph, the students will brainstorm which graph is appropriate. In this activity, a line graph is readable and shows the difference of each tone (shown above).

4. To make the activity more interesting is to find the “mean” (example 1) of each tuning and to discuss the different tuning. Finding the “mean” will enhance the student to identify the different patterns of frequency and improvements on arithmetic functions.

Example 1:

◊ Tuning C = 307.25 Hz
◊ Tuning G = 265 Hz
◊ Tuning D = 344 Hz
Also, the students will have a chance to answer the following question on the activity sheet. These questions will reflect how Math connects to music and its connection outside the classrooms.

5. Finally, have the teacher to explain the data thoroughly and have a 15-30 minute answer discussion as a class.

Part 4: Conclusion

Music and mathematics are closely connected in many ways such as tone, tuning, frequency, rhythm, and pitch (Schmidt Jones, p. 2). This lesson plan integrates mathematical knowledge and fundamentals of culture. Music inspires students with confidence and empowers their intellectual abilities. By introducing students’ cultural heritage it will give them positive outcomes in life and the importance of the Hawaiian culture. Doing hands on activities it will show the student to examine, analyze, recognize, and incorporate data without hesitation. In conclusion, the end results will assist students to internalize their understanding of ethnomathematics and encourage them to ponder the uniqueness of mathematics.

References

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Ukulele Activity Sheet

Directions:

1. Pair up with a partner
   - 1st person pluck every string once while the 2nd person records the data and writes it down on Table 1: "C" Tuning
   - Once the 1st person finishes, switch responsibilities and have the 2nd person repeat the same process while the 1st person records data and writes it down on Table 1: "G" Tuning
   - Again switch responsibilities and record + write down data on Table 1: "D" Tuning

2. Draw graphs and connect points (Each person draws their own graphs)

3. Answer the following questions:
   - Which String has the highest frequency and at what tuning?
   - Which String has the lowest frequency and at what tuning?
   - Find the Mean of each Tuning C, G, & D separately and analyze
     - Tuning C =
     - Tuning G =
     - Tuning D =
   - What did you notice among the 3 tunings? Were there any surprises?
   - Now that you found the frequency of each chord and the Mean, how & why is Mathematics important in music?

4. Discuss the answers as a class for 15-30 minutes

Table 1. Ukulele Frequency

<table>
<thead>
<tr>
<th></th>
<th>&quot;C&quot; Tuning</th>
<th>&quot;G&quot; Tuning</th>
<th>&quot;D&quot; Tuning</th>
</tr>
</thead>
<tbody>
<tr>
<td>First String</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Second String</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Third String</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fourth String</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Section Three: University Level
Part 1: Introduction and History

This lesson plan will teach about addition of vectors with application in early Polynesian navigational practices. It is based in ethnomathematics because it demonstrates how mathematics is related to the local culture, history, and environment (D'Ambrosio, 2001, p. 8).

Archaeologists believe that the Polynesians were the first inhabitants of Hawai'i and arrived around A.D. 500 (Finney, 1979). Over a quarter million Polynesians were living in Hawai'i's eight main islands when Captain James Cook arrived in Hawai'i, in 1778. The Polynesians traveled all over the South Pacific in voyaging canoes to places such as Samoa, New Zealand, and Rapa Nui. The Polynesians were outstanding seafarers. They discovered and created colonies on almost every island in the South Pacific. Without the use of any instruments, the Polynesians were able to navigate through hundreds and even thousands of miles of open ocean to discover new lands. The Polynesians developed colonies on the newly discovered islands. The voyaging canoes were used to transport people, food, water, and domesticated plants and animals since many of the islands were resource-poor. As a result of Polynesian voyaging, the Polynesian islands were united.

During the colonization period, compasses, navigational charts, the spherical coordinate system of latitude and longitude did not exist. The Polynesians were highly skilled at using the stars, sun, moon, ocean swells, and migratory patterns of birds to navigate through the vast ocean (Finney, et al, 1986). At night, the stars were used to determine location because they hold their position in the sky. There was no written language at the time, but the Polynesians used a conceptual star compass to determine direction through the rising and setting of different stars. During the day, close observation of the sun, steady wind patterns and ocean swells were used to navigate. Navigators understood many different swell patterns. In the Pacific, trade winds create ocean swells in predictable east, northeast, and southeast patterns. In order to find land, the navigators looked for cloud and swell patterns, as well as for birds that usually fly up to 20 to 30 miles from land.
The voyaging canoes were composed of two similar hulls that ran parallel to each other (Haddon, 1936). This type of canoe was called a double-hulled canoe (wa’a kaulua), and could reach up to lengths of 60 to 100 feet long. Each hull was made from a hollowed out tree trunk. The two hulls were connected by transversely placed booms (iako). Having two hulls allowed for increased stability, storage and passenger capacity. Thirty or more people could ride on the platform between the hulls. The vessels had either one or two sails made of hala leaves and were attached to the mast and boom. Voyaging canoes were powered by paddling and sails catching wind, and steered with heavy steering blades or moving the position of sails.

Since 1973, the Polynesian Voyaging Society (PVS) in Honolulu has been educating the people of Hawai‘i about Hawai‘i’s seafaring heritage (Lindo, 1980). Through research and repeated non-instrumental voyages, PVS perpetuates the art of non-instrumental navigation in Hawai‘i and has proved that the Polynesians were highly skilled navigators who did not discover new lands by accident. In 1974, the Hōkūle‘a was built (Finney, 1979). Hōkūle‘a is a 62-foot long double-hulled voyaging canoe, modeled after the traditional Polynesian voyaging canoes. Although the design was traditional, both natural and synthetic materials were used in order to have the Hōkūle‘a be well built and maintainable for years to come.

Very few people understood the traditional method of non-instrumental navigation, and it was considered as a vanishing art. However, Mau Piailug, a navigator from Satawal Island in Micronesia proved that he was able to navigate a voyage from Hawai‘i to Tahiti in 1976. In 1979, Mau began training a pupil, Nainoa Thompson (Finney et al, 1986). Through intensive training with Mau, Nainoa successfully guided the Hōkūle‘a from Hawai‘i to Tahiti and back, using non-instrumental navigation. The Polynesian navigators had an intuitive sense of math. Math was needed to estimate speed and time, and to observe the different angles of the stars, planets, and sun. As demonstrated in this lesson plan, math was also needed to understand the different forces that altered the path of the voyaging canoes.

The lesson plan is broken up into four parts. Part one of the lesson plan includes the introduction and history of voyaging canoes and navigation, part two the goal of the lesson plan. Part three covers the methodology of navigation with environmental forces and vectors. Part four is the conclusion.

Part 2: Goal of Lesson Plan

The goal of this lesson plan is to demonstrate how vector addition is related to navigating a voyaging canoe. Following this lesson, students will be able to: 1) define what a vector is, 2) explain how vectors are related to early Polynesian voyaging, 3) find the result of adding two vectors, and 4) create a vector diagram to show the environmental forces on a voyaging canoe.

Part 3: Methodology

Vectors are directed line segments with both direction and magnitude (size). The Latin meaning of the word vector means “carrier” (Merriam-Webster, 2011). It is what carries point A to point B, and is denoted as \( \overrightarrow{AB} \). Vectors are represented graphically with arrows. Arrows should be drawn to scale, so that a vector with a larger magnitude will be represented as a longer arrow than a vector with a smaller magnitude. If a quantity or line segment has only a magnitude, and no direction, it is called a scalar.

A vector can represent velocity because velocity describes both speed and direction. It may also represent a directed distance, or displacement. Vectors can be used to help determine velocity and displacement of a voyaging canoe. In the ocean there are many environmental forces that affect the path of a voyaging canoe, such as wind, ocean swells, and current, which have varying magnitudes and directions. The unit of speed used for wind, current, and boat speed is a knot. One knot is equal to one nautical mile (1.852 km) per hour.

Vector Addition

Vector addition is the addition of two or more vectors resulting in a vector sum. How to use the parallelogram principle: Take two vectors, \( \overrightarrow{A} \) and \( \overrightarrow{B} \). The parallelogram is created by connecting the tails of \( \overrightarrow{A} \) and \( \overrightarrow{B} \). Then, make a copy of \( \overrightarrow{B} \) and connect the tail of it to the head of \( \overrightarrow{A} \). Then finally, make a copy of \( \overrightarrow{A} \), and connect the tail of it to the head of the original \( \overrightarrow{B} \). The addition of two vectors, \( \overrightarrow{A} \) and \( \overrightarrow{B} \), is equal to the diagonal vector \( \overrightarrow{AB} \) of the
You do not need to start at \((0,0)\). If you add the same two vectors at different starting coordinates, the resultant vectors will be identical. The resultant vectors have identical magnitude and direction.

Examples:

7. If a voyaging canoe is traveling at 1 knot, but is not moving, what could explain this?
Answer: The sum of vectors cancels out the velocity of the voyaging canoe. Another possibility is that the current, or wind is traveling in the opposing direction at 1 knot.

8. If a voyaging canoe travels at 4 knots in a directly upstream current moving at 0.4 knots, what is the velocity of the voyaging canoe? Draw a vector diagram.
Answer: See diagram below. The problem is solved by vector addition.

4 knots upstream - 0.4 knots downstream = 3.6 knots upstream.

9. Create a vector diagram in order to calculate the velocity of a voyaging canoe traveling due north at 5 knots, with a westward wind traveling at 10 knots.
Answer:

In order to solve for \(x\), use the Pythagorean theorem \(A^2 + B^2 = C^2\):
\[
\begin{align*}
x^2 &= 5^2 + 10^2 \\
x^2 &= 25 + 100 \\
x^2 &= 125 \\
x &= \sqrt{125} \\
x &\approx 11.18 \text{ knots}
\end{align*}
\]
Therefore, the voyaging canoe will be traveling at 11.18 knots, in the NW direction.
10. A voyaging canoe traveled 8 nautical miles due East, then 6 nautical miles due North. Draw a vector diagram to represent the displacement. What is the total displacement of the voyaging canoe?

Answer:

To find the total displacement of the voyaging canoe, use the Pythagorean theorem $A^2 + B^2 = C^2$.

\[
\text{Displacement}^2 = 6^2 + 8^2 = 36 + 64 = 100
\]
\[
\text{Displacement} = \sqrt{100} = 10 \text{ nautical miles, northeast}
\]

11. The wind is blowing from the Southeast at 15 knots, and you want to sail in a voyaging canoe to a point that is 1 nautical mile away and due North. Create a vector diagram to estimate which direction you should direct your canoe.

Answer: Here is a diagram showing the information given:

In order to get to the destination, we want to think about how to get the sum of vectors to equal a vector that carries the voyaging canoe from starting point to the destination. Remember that vectors have no set origin.

This is a rough diagram, made to help visualize the effect of wind, and the placement of canoe and destination. We are unable to truly add the vectors because they represent different magnitudes – velocity and displacement.

Part 4: Conclusion

Even though the early navigators never took a math course in their entire lives, they were natural mathematicians. As demonstrated by this lesson plan, the concept of vector addition was an integral part in navigating voyaging canoes, especially without the use of navigational instruments. This lesson plan is intended to introduce vectors, however the lesson can be adapted to include trigonometry for the more advanced students. There are many field trips that can be taken in Hawai‘i to enhance the study of vectors in the real world, such as visiting the Polynesian Voyaging Society to learn about the Hōkūle‘a and traditional non-instrumental navigation, six-man outrigger canoe paddling to feel the effects of current and wind, or visiting the Pali Lookout to study the wind. It would be highly beneficial and memorable for the students to experience and explore their culture through mathematics.

References


Part 1: Introduction and History

Hoʻopaʻanaʻau or “memorization” (figuratively) represents a commonly-held value within both Hawaiian and Western intellectual traditions (H. Akaka, personal communication, April 27, 2011). For example, kilolani (expert navigators) must memorize the position of as many stars as possible, so that on cloudy nights (or when only parts of the sky are visible), they can recognize isolated stars or star groups and, thus, imagine the rest of the celestial sphere around them (Mitchell, 1992; Thompson, 2005). Instead of carrying a magnetic “compass,” kilolani hold a mental model of where islands are located and the star points with which to navigate between them (Thompson, 2005). This mental model often takes years of study to build, but with the help of chants, stories, and even dances, navigators can recall complex relationships of geography and location (Thompson, 2005). Mnemonics or “devices used as aids in remembering” (“Mnemonic,” n.d.) can also prove to be useful when making complicated mathematical computations that consistently rely on an established data set (e.g., the trigonometric identities represented within a unit circle). The following lesson challenges students to establish connections between Nainoa Thompson’s star compass and the unit circle, in hopes of helping them more readily hoʻopaʻanaʻau the ʻike (knowledge) contained in both and, thus, be able to complete higher level navigational and trigonometric tasks.

Among Pacific island peoples, culture and ritual were once inseparable from traditional ocean navigator training (Thompson, 2005). Potential candidates were selected with great discretion, in order to ensure that the knowledge surrounding the practice had the greatest chance of survival via these individuals (Thompson, 2005). Thus, a master navigator’s rank was often equal or superior to that of a village chief (Thompson, 2005). Like mathematics navigation relies on no single technique: a navigator determines the position of his or her canoe based
The Hawaiian Star Compass and the Unit Circle

on multiple inputs, which include observations of the seas, skies, and stars as well as memorized knowledge of star, swell, and wind patterns (Thompson, 2005). As a young navigation student, Nainoa derived his own learning tool: a star compass, which shares many similarities with the star compass (from Satawal) that his mentor Mau used to teach navigation.

Mau Piailug, a master navigator from Satawal (Micronesia), had been trained by his grandfather and eventually trained Nainoa Thompson and others to become modern-day kilolani (Thompson, 2005). Nainoa positions the waʻa (canoe) centrally within the compass with the outer circular formation representing the horizon (Thompson, 2005). The right half (the eastern side) of the circle denotes stars' rising points on the horizon, while the left half (the western side) depicts their setting points (Thompson, 2005). In order to orient the waʻa to the rising and setting points of stars, the kilolani uses the 32 equidistant directional points around the horizon (Thompson, 2005). Each point is the midpoint of a “house” of the same name, and each house is 11.25 degrees wide, since 11.25 degrees x 32 houses = 360 degrees (Thompson, 2005).

The four cardinal directions have the following names: east is Hikina (where the sun and stars “arrive” at the horizon), west is Komohana (where the sun and stars “enter” into the horizon), north is ʻĀkau, and south is Hema (Thompson, 2005). These four directional points divide the horizon into four quadrants, which have been associated with wind directions (Thompson, 2005). Northeast is Koʻolau, named for the direction from which the northeast trade winds (the most constant of the Hawaiian winds) blow (Thompson, 2005). Southeast is Malanai, named for a “gentle breeze” associated with Kailua, Oʻahu and Koloa, Kauaʻi, which are both located on the southeastern portions of their respective islands (Thompson, 2005). Southwest is Kona, named for the winds blowing from the south or southwest (Thompson, 2005). Northwest is Hoʻolua, named for a strong north wind that is generated by storm systems passing north of the islands (Thompson, 2005).

In order to orient the waʻa to the rising and setting points of stars, the kilolani uses the 32 equidistant directional points around the horizon, each of which is the midpoint of a “house” spanning 11.25 degrees, i.e., 360 degrees divided by 32 (Thompson, 2005). The following seven houses repeat within each quadrant (Thompson, 2005):

◊ On either side of Hikina (east) and Komohana (west) is Lā (“Sun”), since the sun stays in this house for most of the year as it moves back and forth between its southern limit at the Tropic of Capricorn (23.5 degrees S) at Winter Solstice to its northern limit at the Tropic of Cancer (23.5 degrees N) at Summer Solstice;

◊ Next isʻĀina (“Land”), between 17 degrees and 28 degrees from Hikina and Komohana and can be remembered because the latitudes of Hawaiʻi and Tahiti (both important ʻāina for ancient and modern Polynesian voyagers) are both within this range of degrees;

◊ Noio is a type of bird (i.e., the Hawaiian tern) that helps a kilolani find land because it can be observed flying out to sea (within a radius of 40 miles of an island) in the morning to fish and returning in the evening to rest;

◊ The for houses of Manu (“Bird”) are midway between the four cardinal directions along the horizon, aligning with the beak, tail, and outstretched wing-tips of the centrally pictured bird, the traditional Polynesian metaphor for the waʻa;

◊ Nālani (“The heavens” or “The very high chiefs”) is named for the brightest star in this house, Ke Aliʻi o Kona i Ka Lewa (“The Chief of the South Heavens”) or Canopus;

◊ Nā Leo (“The Voices”) refers to the voices of the stars or kūpuna (ancestors) guiding the kilolani.

◊ Haka (“Empty”) is named for the relative emptiness (of stars, that is) on either side of the houses of ʻĀkau (north) and Hema (south).

A star that rises in a house on the northeast horizon travels across the sky to set in a house of the same name on the northwest horizon, and likewise, a star that rises in a house on the southeast horizon sets in a house of the same name on the southwest horizon (Thompson, 2005). With the rising and setting points of stars are clues to direction, recognizing a star as it rises or sets and knowing the house in which it
rises or sets gives the kilolani the directional point by which to orient the wa’a (Thompson, 2005). Ocean swells are also used to hold a course since they travel from one house on the horizon to a house directly opposite on the horizon (180 degrees away), passing under the wa’a or the center of the compass (Thompson, 2005).

In trigonometry a “unit circle” is a circle with a radius equal to one unit and is centered at the origin (0, 0) in the Cartesian coordinate system in the Euclidean plane. If \((x, y)\) is a point on the unit circle in the first quadrant, then \(x\) and \(y\) are the lengths of the legs of a right triangle whose hypotenuse has length 1. Thus, by the Pythagorean Theorem, \(x\) and \(y\) satisfy the equation \(x^2 + y^2 = 1\) . Special “angle-based triangles” inscribed in a unit circle are useful for visualizing and remembering trigonometric functions with angles of 30 and 45 degrees.

### Part 2: Goal of Lesson

The goal of this lesson plan is to focus on the Mathematics Common Core State Standard MAT. 5.3. Properties and Relationships.

#### Short-term Goals

- To recognize relationships between the cardinal directions, quadrants (wind directions), and “houses” of the Hawaiian star compass and the trigonometric functions of benchmark angles (as well as other angles that are multiples of 30 and 45 degrees) on the unit circle.
- To devise a mnemonic (e.g., a dance, a chant, a story) that would aid in the retention of this basic information about both Nainoa’s star compass and the unit circle.

#### Long-term Goals

- To be able to calculate additional trigonometric values using various operations (e.g., Sum and Difference formulae, Double- and Half-angle formulae, etc.) without having to rely on a visual representation of a unit circle.
- To be able to anticipate star patterns within the night sky and make navigational predictions based on observations.

### Part 3: Methodology

1. Students would individually reflect upon the following questions:
   a. Why might a kilolani receive equal or greater respect than a chief?
   b. How does navigation play a role in your daily life? How do you “navigate” (either literally or figuratively)?

2. Next, the class would be divided into hui of 3-4 students. Each hui would be given an enlarged copy of both the Hawaiian star compass and the unit circle.
3. Together, the students of each hui would be given 20 minutes to brainstorm as many differences, similarities, and/or connections between the two information systems as possible.

4. Each hui would take turns presenting their manaʻo (thoughts) to the rest of their classmates while also compiling others’ manaʻo with that of their own hui.

5. Students would then be given the options of working individually, in pairs, or in their same hui to develop a mnemonic (e.g., a story, a dance, a chant/song) that relates the information contained within Nainoa’s star compass to the information within the unit circle.

6. Mnemonics would be presented during the following class meeting. Assessment would be based on apparent utility of mnemonics for individual students during subsequent assignments.

Part 4: Conclusion

This lesson was designed in response to Nainoa Thompson’s urgent call for educators and knowledgeable cultural practitioners to find ways to connect traditional and modern knowledge, so that math and science can hold relevancy to young Native Hawaiian students (Thompson, N., Personal Communication, May 28, 2011). Ideally, lessons that incorporate multiple ways of knowing will empower students with confidence in both the traditional ʻike of their cultural heritage as well as their own personal intellectual abilities, helping them “navigate” towards more positive outcomes in school and in life.

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Part 1: Introduction and History

For generations, Polynesian voyagers plied the vast distances of the Pacific Ocean in *wa’a kaulua*, or double hulled, voyaging canoes. Though tiny compared to other civilization’s seafaring ships, these voyaging canoes enabled the Polynesians to spread across an unprecedented area. Through innate knowledge of mathematical concepts, the Polynesians built and navigated their vessels with exceeding precision. Though the ancient knowledge faded in recent centuries, there are organizations seeking to revive the Polynesian seafaring tradition. The founder of the Polynesian Voyaging Society wrote in his chronicle of the Society that within a decade of its launch, their first voyaging canoe, Hōkūleʻa, “had proven to be a magnificent vehicle for recovering lost knowledge, and had become a beloved cultural icon in the process” (Finney, 2003, p. 7). Since 1976, Hōkūleʻa has made many successful voyages while being navigated without direction-finding instruments, and it is now joined by newer canoes from island nations all across the far-flung Polynesian triangle. It is important to note, however, that mathematical concepts and applications are not limited to the navigation or design of voyaging canoes. Every day, the sailors who crew the now-numerous voyaging canoes in the Pacific make conscious and unconscious mathematical calculations, and those calculations are important in making vital decisions over the course of a single day. Some of these important calculations involve...
knowledge of trigonometric ratios. An example of this is
the unfortunate and dangerous situation of having a sailor
fall off of the ship, a person-over-board (p.o.b.) scenario.

To establish the situation, we must first cover proper
man-overboard procedures. The Hōkūleʻa’s own p.o.b.
procedure states that the first action of a sailor who has
fallen overboard should be to “Alert the crew. Shout for
help, without swallowing water. Don’t panic” (PVS,
1995, p. 3). In our scenario, however, we assume that
the crew did not notice or hear the sailor in the water.
Following U.S. Coast Guard policy and common sense,
the sailor is wearing a life jacket, but time is of the essence.
The Hōkūleʻa is always trailed on its voyages by an escort
vessel. It is now the sailor’s decision to make which vessel
she should swim toward: Hōkūleʻa or the escort vessel.
She must begin to swim toward one of the ships, and to
swim toward the closest vessel will quicken recovery.
Through the lesson plan, students will solve for the
distance between the sailor and the two vessels using the
trigonometric tangent ratio.

In order to use the tangent ratio to solve for the
adjacent sides, we need to know the adjacent angle and
the height of the masts (the masts are assumed to be at
a right angle to the deck of the vessel). The heights of
the masts are given, and to estimate the adjacent angle,
the sailor’s hand can be used as a laʻau ana (measuring
stick). In pre-contact Hawaiian culture, measuring was
often done with the hand, using various arrangements of
the hand.

According to the 1986 Hawaiian Language
Dictionary by Pukui & Elbert, a half finger width is an
ʻowā. Angles can be measured using the hand because
one ʻowā covers an area of approximately one degree
when the arm is stretched out to its fullest extent. An
easier way to measure, though, is to use a mākahi, or
full finger width. Each mākahi is two ‘owā, or roughly two degrees. This method can be used in configurations of two fingers, a mālua, three fingers, mākolu, and four fingers, māhā. Māhā is useful in either a flat hand or a fist, which together are approximately 8 degrees. From there, measuring in mākahis becomes too difficult, and instead the kīkoʻo and piʻā measures can be used. A kīkoʻo is the distance between the pointer finger and thumb when spread as far apart as possible, in the shape of an “l.” This covers approximately 15 degrees. The full length of the hand, from wrist to fingertip, is one piʻā. An easier way of using the piʻā length is the ubiquitous “shaka.” This is the method described in the lesson worksheet. The “shaka” is useful to estimate angles of 20 degrees, when held vertically (Bushman, 1998). Using these measuring tools, the sailor can quickly and accurately estimate the angle the top and bottom of the vessels’ masts make with her. In combination with the existing knowledge of the mast heights of both Hōkūleʻa and the escort vessel, this will enable her to make the decision of which vessel to swim to with certainty.

This scenario is perfect for applying students’ recently learned trigonometry knowledge in a fun and understandably relevant way. The four problems in this lesson plan all involve using the tangent of an angle to judge distances. Encourage students to discuss each problem openly if they encounter trouble with the calculations. As long as the students are familiar with algebra, they should be able to rearrange any trigonometric function to solve for any side or angle. It is more than possible for the educator to develop further scenarios using the various laʻau ana.

Part 2: Goal of Lesson Plan

Trigonometry can be one of the most useful applications of math, yet it can be confusing at first. This lesson plan seeks to cut away at the confusion by giving a practical application students can understand the relevance of solving right triangles. At the same time, it is important to celebrate the accomplishments of Hōkūleʻa, and its role in the Hawaiian renaissance. By the conclusion of the lesson plan, students should be able to solve a right triangle for the adjacent side. They should also grasp the usefulness of trigonometry outside their classroom. This lesson plan directly meets Common Core State Standard G-SRT.8 - Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. Common Core standards can be viewed at corestandards.org.

Part 3: Methodology

a. (Optional) Before beginning the worksheet, you can familiarize the students with the method of guesstimating angles using a hand by encouraging them to “calibrate” their own hand measurements. Students begin with a fist held at arm’s length directly in front of them. They should be able to “walk” their fists up in front of them, and at the 10th fist should be pointing at the ceiling. Because each fist is the height of 4 fingers (one māhā), and each finger width (mākahi) one is roughly equivalent to 2 degrees, 10 fists are about 90 degrees.

b. The student worksheet contains a page that briefly reviews the trigonometric ratios of a right triangle, and establishes the scenario. It also includes a table of degree measures matching hand positions to degree estimates and the corresponding tangent values. The tangent values are rounded to the nearest clean fraction, and are not exact. The same page has a sample problem to clarify the process. The second page has three problems, varied to enable students to solidify the learning through repetition.
c. After briefly explaining the sample problem, the teacher should ask the students to attempt problem 1a, which does not require a calculator. This first problem replicates the circumstances of the overboard sailor. Students may round as necessary, but should stay reasonable. Some students may come up with different answers. As long as they come up with logical processes, varying answers are okay. They will find out how far off they were in the next problem.

d. After students have calculated their estimates, they should move on to problem 1b, which requires a calculator capable of finding the tangent of acute angles. To find the exact distances, students cannot round figures, and should use the exact tangent as found on their calculators.

e. The additional three problems are word-only (without diagrams). If students make a mistake on the first problem, locate and correct the mistake before progressing to the second problem, and so on.

Part 4: Conclusion

Students often seek relevance in mathematics to their everyday life. As teachers of mathematics, it is important to provide examples of this relevance as often as possible to prevent a stigma from developing in the students’ minds. Math is relevant and applicable we only need to demonstrate this truth. The Hōkūle‘a is a beacon of culture, one that has brought a tremendous sense of pride to the Hawaiian community. To incorporate Hōkūle‘a into mathematics lesson plans blurs the line between culture, history, and mathematics. Exactly as it should be.

References


Hōkūleʻa Overboard Sailor Worksheet

Trigonometric ratios allow us to calculate unknown measures in right triangles. This exercise will use the tangent ratio. The tangent ratio is written as opposite/adjacent. The tangent of an acute angle is the opposite side divided by the adjacent side. Using algebra, we can rearrange this ratio to solve for the length of a side. To solve for the adjacent side, we need to divide the opposite side by the tangent of angle x. Thus, we need both the length of the opposite side and the tangent of x.

In all of the following problems, the length of the opposite side will be given. This is often the height of an object.

To solve the tangent of x, we first need to know the angle of x. In the following problems, the angles of x will be estimated using the “hand ruler” method. This method relies on the width of one finger held at arm’s length covering an area of roughly 2 degrees. For the first part of each problem that follows, you will need to refer to graph 1, as it shows the degree measure of 6 hand positions, and simplified related tangent values. Solve the first part of the problem by estimating the adjacent distance as quickly as you can. The second part of each problem will require a calculator to find the exact tangent value of each angle.

1. A sailor has unfortunately fallen overboard while the double hulled voyaging canoe Hōkūleʻa is making its way around the world. After attempting to raise the awareness of the rest of the crew to her predicament, the sailor must make a decision as to whether she should swim towards Hōkūleʻa or its escort vessel. The sailor knows that he/she can estimate the adjacent angle by using his/her hand. The sailor quickly estimates the angle the top and bottom of the vessels’ masts make with her as 15 degrees because she knows that when she spreads her fingers in the shape of an “L”, they cover a 15 degree angle. Furthermore, the sailor already knows that Hōkūleʻa’s mast height is 31 feet and the escort vessel Kama Hele’s mast is 42 feet tall. Help the sailor make the decision of which vessel to swim to with certainty by using your knowledge of trigonometry to quickly estimate the distance between her and each of the ships.

The diagram below is to help you picture the situation and fill in variables.

a. What is your estimate for the respective distances between the sailor and the two vessels? Do not use a calculator. Round numbers if necessary to make figuring quicker.

Between the sailor and Hōkūleʻa

Between the sailor and the escort vessel Kama Hele

b. Now that you have an estimate, take your calculator and find the actual distances.

Between the sailor and Hōkūleʻa

Between the sailor and the escort vessel Kama Hele
2. The same unfortunate sailor has again fallen overboard while Hokule'a sails past Diamond Head. Once again, the sailor must use her knowledge to find which direction to swim. This time the two options are to swim towards Hokule'a or the Diamond Head coast. The Diamond Head light house is 55 feet tall and is built on ground 147 feet above sea level. The sailor finds that two fingers cover the height of Hokule'a's mast, and a shaka covers the lighthouse and the cliff below it.

   a. What is your estimate for the respective distances between the sailor and Hokule'a and the sailor and the coast? Do not use a calculator. Round numbers if necessary to make figuring quicker.

Between the sailor and Hokule'a

Between the sailor and the coast

b. Now that you have an estimate, take your calculator and find the actual distances.

Between the sailor and Hokule'a

Between the sailor and the coast

3. A sailor on board Hokule'a is attempting to sail in a narrow passage between two cliffs. The height of the cliff on the left is 100 feet, the cliff on the right is 75 feet. The sailor wishes to remain 200 feet from both cliff faces at all times. If the angle on the left is approximately a shaka, while the right cliff is an 'L' angle, is the sailor accomplishing his goal?

   a. What is your estimate for the respective distances between Hokule'a and both cliff faces? Do not use a calculator. Round numbers if necessary to make figuring quicker.

Between Hokule'a and the right cliff face

Between Hokule'a and the left cliff face

b. Now that you have an estimate, take your calculator and find the actual distances.

Between Hokule'a and the right cliff face

Between Hokule'a and the left cliff face

4. A sailor on the ship Noa Noa falls off the vessel. He is unable to raise awareness of his situation on the vessel. He can see both Hokule'a and Noa Noa in the distance. Which should he swim towards? Noa Noa's mast height is 45 feet. The sailor calculates a three finger angle between him and Hokule'a and a one finger angle with Noa Noa.

   a. What is your estimate for the respective distances between the sailor and the two vessels? Do not use a calculator. Round numbers if necessary to make figuring quicker.

Between the sailor and Hokule'a

Between the sailor and Noa Noa

b. Now that you have an estimate, take your calculator and find the actual distances.

Between the sailor and Hokule'a

Between the sailor and Noa Noa
Part 1: Introduction and History

Art is often a representation of an individual or culture's identity and history. It is a reflection or lens in which the artist viewed, lived, and valued their world. Many cultures have used art as a means of expression and communication that helped to perpetuate their values. The traditional Hawaiian hand-tapped tattoo is one way the ancient Hawaiians used art in their culture. The process of getting one of these tattoos start with multiple protocols and prayers and is considered a sacred rite more than an artwork. Each of these tattoos is unique. The design bestowed is based on the wearer's role in the community and genealogy. Most of the Hawaiian tattoo motifs utilize diamonds, triangles, and curved shapes. Also the negatives of these tattoos form unique geometric patterns. The one thing in common with all of the Hawaiian tattoos is that they contain some type of symmetry.

Students will draw an image that reflects their culture to help them grasp the concept of definite integrals and area between the curves. Having students draw their own images will make math more personal to them and more relatable. It also allows the students to show their creativity with math, which otherwise is uncommon.

Part 2: Goal of Lesson Plan

The goal of this project is to mix the two abstracts, art and mathematics, to give the students, who are learning calculus, a better transition into differentiation and integration, the basis of calculus. The following Mathematics Common Core State Standards are utilized:

$\diamond$ MAC.9.6: Use Riemann sums, the trapezoidal rule, and technology to approximate definite integrals of functions represented algebraically, geometrically, or by tables of values

$\diamond$ MAC.9.7: Find specific antiderivatives using initial conditions, including finding velocity functions from acceleration functions, finding position functions from velocity functions, and applications to motion along a line.
\[
\begin{align*}
\int_a^b f'(x) \, dx &= f(b) - f(a) \\
\int_a^b [f(x) + g(x)] \, dx &= \int_a^b f(x) \, dx + \int_a^b g(x) \, dx \\
\int_a^b k \cdot f(x) \, dx &= k \int_a^b f(x) \, dx \\
\int_a^b f(x) \, dx &= 0 \\
\int_a^b f(x) \, dx &= -\int_a^b f(x) \, dx \\
\int_a^b f(x) \, dx + \int_b^a f(x) \, dx &= \int_b^a f(x) \, dx
\end{align*}
\]

If \( f(x) \leq g(x) \) on \( [a, b] \), then \( \int_a^b f(x) \, dx \leq \int_a^b g(x) \, dx \)

◊ MAC.9.8: Use definite integrals to find the area between a curve and the x-axis, the average value of a function over a closed interval, and the volume of a solid with known cross-sectional area.

◊ MAC.10.5: Apply the fundamental theorem of calculus; i.e., interpret a definite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval, that is

\[
\int_a^b f'(x) \, dx = f(b) - f(a)
\]

◊ MAC.10.6: The student: Uses the following properties of definite integrals to evaluate definite integrals:

\[
\begin{align*}
\int_a^b [f(x) + g(x)] \, dx &= \int_a^b f(x) \, dx + \int_a^b g(x) \, dx \\
\int_a^b k \cdot f(x) \, dx &= k \int_a^b f(x) \, dx \\
\int_a^b f(x) \, dx &= 0 \\
\int_a^b f(x) \, dx &= -\int_a^b f(x) \, dx \\
\int_a^b f(x) \, dx + \int_b^a f(x) \, dx &= \int_b^a f(x) \, dx
\end{align*}
\]

If \( f(x) \leq g(x) \) on \( [a, b] \), then \( \int_a^b f(x) \, dx \leq \int_a^b g(x) \, dx \)

◊ MAC.10.10: Find average and instantaneous rates of change

Part 3: Methodology

◊ Practice with a few basic combinations of functions to form obvious images/symbols (each of which the students can choose, that increase in complexity as they progress).

◊ Students should choose images that reflect their background/interests.

◊ Draw image free hand.

◊ Draw image on graph paper, plot the image on paper, using functions to describe every mark made (Must incorporate trig functions, polynomials, and logarithmic functions).

◊ Find the derivatives of 10 of the functions used.

◊ Approximate the area of the image’s silhouette using Riemann Sum and trapezoidal rule.

◊ Now find the actual area of your image’s silhouette using definite integrals.

First, to get the students comfortable with creating images with functions, get them to graph their initials or a single letter. You may want to do a slight review on finding lines of best fit with the students.

Remember that the students final graphs should be drawn free handed, but they can use online tools/calculators to play with which functions to use.

In this example, I used two functions to create the right side of an M and then I reflected those about the y axis to create the other half.

This is the M with the restrictions on the domain.
Next the students should create something slightly more advanced, mixing line functions with a polynomial, to make a very simple image.

Then the students should add restrictions on the domains of the functions they’ve chosen so that the image is clear.

Now the students should try something a bit more challenging like a face, symbol, etc. In this example I used three quadratics to create part of a dog.

I added in the restrictions for the first three functions where they equal each other. To keep the image symmetric, I took the function \( f(x) = 10(x-2)^2 - 2 \) and shifted it to the left by 4.

Adding in the restrictions where the previously added in function meets the other functions, we get the following:

To make the image more defined, I added in conics for the eyes, nose, and tongue.

Adding in restrictions for the nose and tongue, we get the following:

When the students are done they can color in their image to make it more aesthetically pleasing, but since it’s just for practice the students should only add detail if they want to.

Now that the students have a better idea of how this works, they will work on their final image (This is the one that they will be using calculus to find the area). This one must incorporate logarithmic functions, trigonometric functions, and polynomials. The functions will be the ones that the students will be differentiating and integrating, so make sure all of the functions used are written down and organized.

**Reminder:** the students final graphs should be drawn free handed, but they can use online tools/calculators to play with which functions to use.
Here is an example of how the students should set it up. This image is the setup of the top of a honu.

Next I worked on the feet for the honu.

Here are all the functions used (without restrictions).

Now the Students should break the silhouette of their image up into segments where the top and bottom functions differ.

Then the students will need to find the area of each segment, using Riemann Sum, trapezoidal rule, and definite integrals. Finally, to find the total area of their image, the students will need to do is find the sum of each segments area.

Overall, the students should:

◊ Have three versions of their image, their first drawing, graph and functions without restrictions, and the graph with functions and restrictions broken up into segments, all to be done by hand. The students may use their graphing calculators to find the functions they want to use.

◊ Choose 10 of their functions and tell whether or not the function is concave up and/or down on the interval of the image (must include at least one trigonometric and logarithmic function).

◊ Find the area of their image using both the Riemann sum and trapezoidal rule.

◊ Find the area of the silhouette of their image using definite integrals. Students may use technology to double check their results.

Part 4: Conclusion

Calculus is a milestone in any student’s educational career. This lesson plan will, hopefully, give students a smooth transition from trig/pre-calculus into the concepts of calculus. This lesson plan is meant to follow the student throughout their first semester calculus class so that they aren’t just looking at the functions and graphs in their textbook, but functions and graphs that they came up with.
Hawai‘i, our island home, is a place where we strive to honor and respect people, values, and heritages. There has never been a more important time to cultivate sustainable conditions that advance student success in college. According to the National Council of Teachers of Mathematics (NCTM) Position Statement on “Equity in Mathematics Education” (2008):

A culture of equity depends on the joint efforts of all participants in the community of students, educators, families, and policymakers… High expectations, culturally relevant practices, ethnomathematics, and attitudes that are free of bias, and unprejudiced beliefs expand and maximize the potential for learning… All students should have access to and engage in challenging, rigorous, and meaningful mathematical experiences (pp. 1-2).

Through the Ethnomathematics Summer Institute, University of Hawai‘i mathematics faculty, staff, and students designed and implemented mathematics lessons grounded in the ethnic, socioeconomic, historical, and cultural diversities of our state. We have learned about the importance of fostering an environment of critical thinking, honing quantitative literacy skills, and furthering the development of quality mathematics education.

Whether your journey is at the local, state, or national level, it is our hope that we can work together to be catalysts for positive change. When the inventions, experiences, and applications of mathematics of all students are realized and respected, we provide them with equal opportunity for access and achievement.

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Students and faculty learn about biacoustics, sustainability, and trigonometry while at the Hawai‘i Institute of Marine Biology, May 2012.


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