WORKSHEET 9

(1) In the following problems, you should write the partial fraction decomposition for the rational function, but do not solve for the constants. For example, for $\frac{x}{(x^2-1)(x^2+1)}$, you should write

$$\frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$$

because the denominator factors as $(x-1)(x+1)(x^2+1)$.

(a) $\dfrac{x-2}{x^2+4x}$

(b) $\dfrac{x^2}{x-1}$ (Be careful! The numerator has higher degree than the denominator.)

(c) $\dfrac{x^2+1}{x^2-9x-10}$

(d) $\dfrac{x^2+2}{x^2(x^2+1)^2(x-3)^3}$

(2) Put the following into partial fractions form:

(a) $\dfrac{2x^2 - 5}{x^2 - 3x - 4}$

(b) $\dfrac{3x + 1}{x^3 + 2x^2 + 4x}$

(3) Integrate the following functions (no need to be put in partial fractions form):

(a) $\displaystyle\int \frac{1}{y^2 - 2y + 4}\mathrm{d}y$

(b) $\displaystyle\int \frac{3x + 2}{2x^2 - 4x + 8}\mathrm{d}x$

(4) Put the following functions in partial fractions form and integrate

(a) $\displaystyle\int_4^8 \frac{y}{y^2 - 2y - 3} \mathrm{d}y$

(b) $\displaystyle\int \frac{x^2 + x}{x^3 + x} \mathrm{d}x$

(5) Find an upper bound for the Trapezoid rule error with $n = 4$ when approximating

$$\int_{-1}^{1} e^{-x^2}$$

You don't need to find the approximation, only the upper bound for the estimate. You may find this helpful:

$$|E_T| \leq \frac{M(b-a)^3}{12n^2}, \quad \text{where } |f''(x)| \leq M \text{ for all } x \text{ in } [a,b]$$

(6) Find an n that would guarantee that Simpson's Rule S_n is within 10^{-8} of $\int_1^4 x^{3/2}\mathrm{d}x$. You may leave your answer abstractly with roots. You can use the following bit of information $\frac{d^4}{dx^4}(x^{3/2}) = \frac{9}{16}x^{-5/2}$. **Again: you don't need to find the approximation.** You may find this helpful:

$$|E_S| \leq \frac{M(b-a)^5}{180n^4}, \quad \text{where } |f^{(4)}(x)| \leq M \text{ for all } x \text{ in } [a,b]$$

(7) For the following problems explain why the integral is improper. Evaluate or explain why it is divergent.

(a) $\displaystyle\int_0^1 \frac{\mathrm{d}x}{x^{3/2}}$

(b) $\displaystyle\int_{-\infty}^0 \theta e^{-\theta}\mathrm{d}\theta$

(8) For the following problems, explain whether the integral is divergent or convergent. You do not have to evaluate. Try to use the p-test if you can.

(a) $\displaystyle\int_1^\infty \frac{1+\sin x}{x^2}\,\mathrm{d}x$

(b) $\displaystyle\int_1^\infty \frac{\sqrt{x^4+1}}{x^3}\,\mathrm{d}x$

(c) $\displaystyle\int_1^\infty \frac{\arctan x}{x}\,\mathrm{d}x$