Name: ______________________________

In many cases, you have to just circle an answer.
Please, be careful, I will take points off for wrong circles!

Question 1

Circle True/False

TRUE	FALSE	Every group of order n has an element of order n
TRUE	FALSE	The groups $\mathbb{Q}$ and $\mathbb{Z}$ (both additive) are isomorphic
TRUE	FALSE	The groups $\mathbb{Q}^*/\{1,-1\}$ and $\mathbb{Q}^{**}$ are isomorphic
TRUE	FALSE	The groups $\mathbb{R}$ and $\mathbb{R}^{**}$ are isomorphic
TRUE	FALSE	The groups $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ and $\mathbb{Z}_4$ are isomorphic
TRUE	FALSE	The groups $\mathbb{Z}_2 \oplus \mathbb{Z}_3$ and $\mathbb{Z}_6$ are isomorphic
TRUE	FALSE	The groups $\mathbb{C}^*$ and $\mathbb{R}^{**} \times \mathbb{R}/\mathbb{Z}$ are isomorphic
TRUE	FALSE	The groups $\mathbb{C}^*$ and $\mathbb{R}^* \times \mathbb{R}^*$ are isomorphic
TRUE	FALSE	For $n \geq 4$, the group S_n is not simple
TRUE	FALSE	Let $n \geq 3$. For any permutation $\sigma \in S_n$ the permutation σ^2 is even

Question 2

Let G be a group, and assume that $G \neq \{e_G\}$.

Circle True/False

TRUE	FALSE	If G contains an element of infinite order, then G is infinite
TRUE	FALSE	If G contains no elements of infinite order, then G is finite
TRUE	FALSE	If G is simple, then $Z(G) =< e >$
TRUE	FALSE	If G is simple and G is not abelian then $Z(G) =< e >$
TRUE	FALSE	If G has a subgroup $H \neq \{e_G\}$ of index $[G:H] = 2$, then G is not simple

Question 3

In this question, count only **non-trivial proper** subgroups. Specifically, please exclude the whole group and $\{(0,0)\}$ from the counting.

a) How many proper subgroups does the group $\mathbb{Z}_7 \times \mathbb{Z}_7$ have?

Answer

b) How many proper subgroups does the group $\mathbb{Z}_{11} \times \mathbb{Z}_7$ have?

Answer

Question 4

a) How many non-isomorphic abelian groups of order 81 are there?

b) How many non-isomorphic abelian groups of order 105 are there?

Question 5

What is the order of A_4?

Question 6

Let

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 2 & 8 & 9 & 4 & 5 & 7 & 1 & 6 \end{pmatrix} \in S_9$$

a) Find σ^{17}.
Answer: (your are required to fill in the second row)

$$\sigma^{17} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ & & & & & & & & \\ & & & & & & & & \end{pmatrix}$$

b) Is σ^{17} even or odd?

Circle your answer: EVEN ODD

Question 7

Circle True/False

TRUE FALSE Every group of order 77 is abelian

TRUE FALSE Every group of order 49 is abelian

TRUE FALSE Every group of order 77 is cyclic

TRUE FALSE Every group of order 49 is cyclic

Question 8

Let H_1 be a subgroup of G_1 and let H_2 be a subgroup of G_2, and let p be a prime (both G_1 and G_2 are assumed to be finite.)

Please circle TRUE if a statement is always true, and FALSE if a statement may be false.

TRUE FALSE $H_1 \times H_2$ is a subgroup in $G_1 \times G_2$

TRUE FALSE If H_1 is normal in G_1 and H_2 is normal in G_2,
then $H_1 \times H_2$ is a normal subgroup in $G_1 \times G_2$

TRUE FALSE If H_1 is a Sylow p-subgroup in G_1 and H_2 is a Sylow p-subgroup in G_2,
then $H_1 \times H_2$ is a Sylow p-subgroup in $G_1 \times G_2$

TRUE FALSE If H_1 is the unique Sylow p-subgroup in G_1
and H_2 is the unique Sylow p-subgroup in G_2,
then $H_1 \times H_2$ is the unique Sylow p-subgroup in $G_1 \times G_2$