Involution of Λ-adic analytic spaces and the U_p-operator for half-integral weight modular forms

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Abstract. We consider certain natural involution acting on the Λ-adic analytic space associated with the ordinary metaplectic Hecke algebra. We show that this involution is tightly connected with U_p-operator acting on half-integral weight cuspforms.

0. Introduction

Although there already appeared many articles addressing p-adic (and modulo p) theory of modular forms of half-integral weight, the theory looks far from being accomplished. In particular, the important role of U_p-operator was indicated by Jochnowitz [**3**]. The author hopes that the present text sheds some light on the subject. In particular, we connect this operator with a certain natural involution acting on Λ-adic analytic spaces. The setting of this paper is not as general as it might be possible. We consider the setting which seems to be the most interesting one, and just indicate the generalizations which may be obtained without essential modification of the argument.

The basic ingredients of our method are the Kohnen's refinement of Shintani lifting [**6**], and the Stevens' Λ-adic interpolation of cycle integrals [**9**].

The contents of the paper are as follows.

In the first section, we describe the setting, and construct the claimed involution of our Λ-adic analytic spaces. We also formulate and discuss the result (see the theorem below) in this section.

We quote a result from [**9**] in the form appropriate for our consideration in the second section. This concerns the Λ-adic interpolation of cycle integrals.

The formulas for Fourier coefficients of half-integral weight modular forms in the terms of cycle integrals are given in the third section. This is essentially a quotation from [**6**].

The fourth section is devoted to the proof of the theorem.

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1. Statement and discussion of the results Let $p > 5$ be a rational prime. Fix an embedding $\overline{\mathbb{Q}} \hookrightarrow \overline{\mathbb{Q}}_p$. We will identify algebraic numbers and their images under this embedding. Denote by v_p the p-adic valuation on $\overline{\mathbb{Q}}_p$ normalized such

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that $v_p(p) = 1$. Let

$$\Lambda = \mathbb{Z}_p[[1 + p\mathbb{Z}_p]]$$

be the completed group ring on the principal unit group $1 + p\mathbb{Z}_p$. Put

$$\Lambda_1 = \Lambda[(\mathbb{Z}/p\mathbb{Z})^*].$$

For a finite flat Λ-algebra $\mathcal{R}$, put

$$\mathcal{X}(\mathcal{R}) = Hom_{cont}(\mathcal{R}, \overline{\mathbb{Q}}_p)$$

Restriction to Λ induces a surjective finite-to-one mapping

$$\pi : \ \mathcal{X}(\mathcal{R}) \to \mathcal{X}(\Lambda).$$

Using the local sections of π, one defines analytic charts around unramified points. Thus, $\mathcal{X}(\mathcal{R})$ becomes a Λ-adic analytic space [**9**].

Note that $\mathcal{X}(\Lambda_1)$ has the natural group structure (induced by the multiplication in $\overline{\mathbb{Q}}_p^*$) since

$$\mathcal{X}(\Lambda_1) \simeq Hom_{cont}(\mathbb{Z}_p^*, \overline{\mathbb{Q}}_p^*).$$

Denote by ξ the non-trivial quadratic Dirichlet character modulo p. We regard ξ as an element of $\mathcal{X}(\Lambda_1)$:

$$\xi : u \mapsto \xi(u) \qquad \text{for } u \in \mathbb{Z}_p^*.$$

Denote by ι the multiplication by ξ in the group $\mathcal{X}(\Lambda_1)$. Thus, ι is a natural involution of $\mathcal{X}(\Lambda_1)$.

We consider the Λ_1-algebra $\mathcal{R}_1$, which is the universal p-ordinary Hecke algebra of tame conductor 1. Put, as in [**9**],

$$\tilde{\mathcal{R}}_1 = \mathcal{R}_1 \otimes_{\Lambda_1, \sigma} \Lambda_1$$

for the universal ordinary metaplectic Hecke algebra of tame conductor 1. Here σ is the ring homomorphism associated to the group homomorphism $t \mapsto t^2$ on $\mathbb{Z}_{p,1}^*$, which is the inverse limit of $(\mathbb{Z}/p^m\mathbb{Z})^*$ as $m \to \infty$. (Recall that $\Lambda_1 \simeq \mathbb{Z}_p[[\mathbb{Z}_{p,1}^*]]$.) Following [**9**], we regard $\tilde{\mathcal{R}}_1$ as a Λ_1-algebra by equipping it with the structural homomorphism

$$\Lambda_1 \to \tilde{\mathcal{R}}_1 \qquad \lambda \mapsto 1 \otimes \lambda.$$

The ring homomorphism (which is not a homomorphism of Λ_1-algebras)

$$\mathcal{R}_1 \to \tilde{\mathcal{R}}_1 \qquad \alpha \mapsto \alpha \otimes 1$$

induces the map on Λ-adic spaces

$$\mathcal{X}(\tilde{\mathcal{R}}_1) \to \mathcal{X}(\mathcal{R}_1) \tag{1}$$

Let F be a normalized cuspidal p-ordinary Λ-adic Hecke eigenform of tame conductor 1. This is a formal power series $F(X, q) \in \mathcal{O}[[X, q]]$,. For $k \geq 1$, and $2k \equiv 2k_0 \bmod p - 1$, the q-series $F(u^{2k} - 1, q)$ is the q-expansion of a normalized p-stabilized cuspidal Hecke eigenform of level 1. According to [**2**, Proposition 7.2.2], we have

$$F(u^{2k} - 1, q) = f_{2k}^*(\tau) = f_{2k}(\tau) - \beta_{2k} f_{2k}(p\tau) \text{for } k > 1 f_2^*(\tau) = f_2(\tau) \text{for } k = 1 \tag{2}$$

Here f_{2k}, for $k > 1$, is a normalized p-ordinary cusp Hecke eigenform of weight $2k$ on $SL_2(\mathbb{Z})$, and β_{2k} is the root of its Hecke polynomial which is not a p-adic unit, and f_2 for $k = 1$ is a normalized Hecke eigenform of weight 2 on $\Gamma_0(p)$.

The Λ-adic form F corresponds to the decomposition

$$\begin{aligned} \mathcal{R}_1 &= \Lambda_1 \oplus \mathcal{R}_1' \\ \tilde{\mathcal{R}}_1 &= \tilde{\Lambda}_1 \oplus \tilde{\mathcal{R}}_1'. \end{aligned} \tag{3}$$

Let $J \in \tilde{\mathcal{R}}_1$. We denote by the same letter J its image in

$$\tilde{\Lambda}_1 = \Lambda_1 \otimes_{\Lambda_1,\sigma} \Lambda_1$$

under the projection to the first component in (3).

The involution $\iota : \mathcal{X}(\tilde{\Lambda}_1) \longrightarrow \mathcal{X}(\tilde{\Lambda}_1)$ comes from $\mathcal{X}(\Lambda_1) \xrightarrow{\iota} \mathcal{X}(\Lambda_1)$ (multiplication by ξ), and the fact that the structural homomorphism $\Lambda_1 \to \Lambda_1 \otimes_{\Lambda_1,\sigma} \Lambda_1 = \tilde{\Lambda}_1$ with $\alpha \mapsto 1 \otimes \alpha$ is an isomorphism of rings.

We identify an arithmetic point $\kappa \in \mathcal{X}(\Lambda_1)$ of signature (t, χ) with its signature, and write $\kappa = (t, \chi)$.

As usual, we compute the signature of an arithmetic point $\kappa \in \mathcal{X}(\mathcal{R})$ (any finite flat $\mathcal{R}$) with respect to the structural homomorphism $\Lambda_1 \to \mathcal{R}$.

Denote by $\kappa = (k, \chi)$ the arithmetic point of $\mathcal{X}(\tilde{\Lambda}_1)$ which takes $1 \otimes u$ to $u^k \chi(u)$ for $u \in \mathbb{Z}_p^*$. Remark that both the point κ and $\iota(\kappa) = (k, \xi\chi) \in \mathcal{X}(\tilde{\Lambda}_1)$ project by (1) (and (3) restricted to the first component) to the point $(2k, \chi^2) \in \mathcal{X}(\Lambda_1)$.

Let us now restrict our attention to the arithmetic points $\kappa \in \mathcal{X}(\Lambda_1)$ with $\kappa = (2k, id)$, where k is a positive integer, $2k \equiv 2k_0 \bmod p-1$, and $id = id_p$ stands for the trivial Dirichlet character modulo p. The points $\tilde{\kappa} = (k, id)$ and $\iota\tilde{\kappa} = (k, \xi)$ project to κ. There is the normalized cusp Hecke eigenform f_{2k} defined by (2), which corresponds to κ. Consider the cuspform $\phi_k = \sum c_k(n) q^n$ of half-integral weight $k + 1/2$, which belongs to the Kohnen's subspace $S^+_{k+1/2}$ (i.e. $c_k(n) = 0$ unless $(-1)^k n \equiv 0, 1 \bmod 4$, see [**5, 6**]), and is connected to f_{2k} by the Shimura correspondence. It is not identically zero, and is defined up to a constant multiple. We associate this form with the point $\tilde{\kappa} = (k, id)$. Consider the point $\iota\tilde{\kappa}$. The point $\tilde{\kappa}' = (k + \frac{p-1}{2} p^N, id)$ is as close to $\iota\tilde{\kappa}$ as large is $N \geq 0$ (the corresponding characters on $\mathbb{Z}_p^*$ are congruent modulo p^N). Consider the cuspform $\phi_l \in S^+_{l+1/2}$ with $l = k + \frac{p-1}{2} p^N$ associated with $\tilde{\kappa}'$ as above. Naively speaking, the involution ι should take ϕ_k almost to ϕ_l. The purpose of this paper is to clarify this statement.

Alternatively, if $\phi_k \in S^+_{k+1/2}$ for $k > 1$, (or $S^+_{3/2}(p)$ for $k = 1$) is a Hecke eigenform, the U_p-operator which on Fourier expansions is defined by

$$U_p : \sum c(n) q^n \mapsto \sum c(pn) q^n,$$

and acts on ϕ_k almost as an involution. Namely (see (8) below) $U_p^2 \phi_k \equiv \phi_k \bmod p^{k-1}$ for $k > 1$, and $U_p^2 \phi_1 = \phi_1$. Here and in the following a congruence between two forms means that their q-expansion coefficients are congruent.

The present paper reveals the tight connection between these two natural "almost involutions". Roughly speaking, our main result (see the theorem below) claims that U_p-operator interchanges ϕ_k and ϕ_l, as above, modulo a power of p.

Here are some remarks on the setting described so far.

For the sake of transparency we will only start with the point $(k_0, id) \in \mathcal{X}(\tilde{\Lambda}_1)$ with $k_0 = 1$. It seems to the author that this case is the most interesting one. The case of arbitrary $k_0 > 1$ may be considered along the same lines, with certain simplifications.

The argument (and the result) might be generalized to the case of odd square-free tame conductor (cf. [**6**]). However, further direct generalization in this direction does not look possible. Evidently, for higher tame conductors, one has to switch from half-integral weight modular forms to Jacobi forms. In the case of trivial tame conductor, which is considered in the present paper, the language of half-integral weight modular forms is better unless one does not consider the skew-holomorphic Jacobi forms.

Since both the results and the argument of the paper are in fact of local nature (with respect to Λ) one can get rid of the assumption that there exist a (global) Λ-adic form F (cf. [**1**, 2.7]).

We are now ready to formulate our main result.

Theorem *One can choose a (non-zero) normalization for the half-integral weight modular forms ϕ_k for each $k \geq 1$ such that their Fourier coefficients are algebraic integers, and*

$$U_p\phi_1 \equiv \phi_l \bmod p^A$$

if $l \equiv 1 \bmod (p-1)p^{(A-1)}/2$, and $l \not\equiv 1 \bmod p-1$.

Remarks

Since $U_p^2\phi_1 = \phi_1$, the operator just interchanges ϕ_1 and ϕ_l modulo p^A.

It might come out that, under our normalization, all the Fourier coefficients of ϕ_1 are divisible by p^B with some $B > 0$. In this case, the assertion of theorem is vacuous when $A \leq B$. This begins, however, to produce non-trivial congruences when $A > B$.

Example Put $p = 11$. In this case, there is a unique normalized p-ordinary Λ-adic form (cf. [**2**, p.234], and [**1**, Example 2.11]). Write

$$\eta = q^{\frac{1}{24}} \prod_{n\geq 1}(1-q^n)$$

for the classical η-function. In particular, we have the specializations

$$f_2 = \eta(\tau)^2\eta(11\tau)^2,$$

$$f_{12} = \Delta = \eta(\tau)^{24}.$$

Of course, $f_{12} \equiv f_2 \bmod 11$.

Write $\theta = 1 + \sum_{n\in\mathbb{Z}} q^{n^2}$ for the classical theta-function, and $E_{2k} = \frac{B_{2k}}{4k} + \sum_{n\geq 1}\sigma_{2k-1}(n)q^n$ for the Eisenstein series. The half-integral weight cusp forms connected with f_{12} and f_2 via Shimura correspondence are

$$\phi_1 = U_4(\eta(2\tau)\eta(22\tau)\theta(11\tau)) \tag{4}$$

and

$$\phi_6 = (2E_4(\tau)\theta'(\tau) - E_4'(\tau)\theta(\tau)). \tag{5}$$

According to the theorem, one expects that U_{11} connects (4) with (5) modulo 11. Indeed, one can check that

$$5\phi_1 \equiv U_{11}\phi_6 \quad \text{and} \quad 5U_{11}\phi_1 \equiv \phi_6 \bmod 11 \tag{6}$$

The half-integral weight modular forms ϕ_1 and ϕ_6 appeared as examples in the classical papers [**8**], and [**4**]. It turns out that they are connected by (6).

2. p-Adic interpolation of cycle integrals

In this section, we reformulate a result from [**9**] in the form suitable for our consideration.

Write $Q = [a, b, c](x, y)$ for an integral binary quadratic form $ax^2 + bxy + cy^2$ (we allow $a = 0$), and $Q' = [a, -b, c]$. For a cuspform f of weight $2k$ on $\Gamma_0(p)$, and $a \equiv 0 \bmod p$, put

$$j_Q(f) = \int_{C_Q} f(\tau)(a\tau^2 + b\tau + c)^{k-1} d\tau$$

where the cycle C_Q is defined as in [**6**, p.240]. The proposition below is a reformulation of Theorem 5.5 and Lemma 6.1 of [**9**].

Proposition *Let $\tilde{\kappa}_0 = (k_0, id) \in \mathcal{X}(\tilde{\Lambda}_1)$. There exist*
- *complex periods $\Omega^-(\tilde{\kappa}) \in \mathbb{C}^*$*
- *p-adic periods $\Omega_{\tilde{\kappa}} \in \overline{\mathbb{Q}}_p$ with $\Omega_{\tilde{\kappa}_0} \neq 0$*
- *an element $J_Q \in \tilde{\Lambda}_1$ defined for any integral quadratic form $Q = [a, b, c]$ with $a \equiv b \equiv 0 \bmod p$ with the following properties.*

$$J_Q(\tilde{\kappa}) = \frac{1}{2} \frac{\Omega_{\tilde{\kappa}}}{\Omega^-(\tilde{\kappa})} \chi(c)(j_Q(f^*_{2k}) + j_{Q'}(f^*_{2k})) \tag{7}$$

if $\tilde{\kappa} = (k, \chi) \in \mathcal{X}(\tilde{\Lambda}_1)$ lies over $\kappa = (2k, id) \in \mathcal{X}(\Lambda_1)$ with $k \geq 1$.

The odd periods are p-integers and

$$v_p\left(\frac{\Omega_{\tilde{\kappa}}}{\Omega^-(\tilde{\kappa})} \int_0^{i\infty} f_{2k}(\tau)\tau^s d\tau\right) \geq 0$$

if $s = 0, 2, \ldots, 2k-2$.

Notice that the restriction to quadratic or identity characters χ in the above proposition comes just from our restriction to the points $\kappa = (2k, id) \in \mathcal{X}(\Lambda_1)$.

3. Fourier coefficients of half-integral weight cuspforms d'aprés W. Kohnen

We make use of the connection between cycle integrals and Fourier coefficients of certain (non-zero) cuspforms of half-integral weight established by Kohnen [**6**]. Now recall some results from loc. cit..

The group $SL_2(\mathbb{Z})$ acts on integral binary quadratic forms by

$$[a, b, c]\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}(x, y) = [a, b, c](\alpha x + \beta y, \gamma x + \delta y).$$

The number of equivalence classes of quadratic forms with fixed discriminant is finite. If $D \equiv 0, 1 \bmod 4$ is an integer and $Q = [a, b, c]$ is a form whose discriminant $|Q|$ is divisible by D, let

$$\omega_D(Q) = 0 \text{if } (a, b, c, D) > 1 \left(\frac{D}{r}\right) \text{ if } (a, b, c, D) = 1, \text{ and } Q \text{ represents } r \text{ with } (r, D) = 1.$$

Write

$$\phi_k = \sum_{n>0} c_k(n) q^n$$

for the non-zero modular form of weight $k+1/2$ which belongs to the "+"-subspace, and is connected to f_{2k} via the Shimura correspondence. In particular, for a positive integer n such that $(-1)^k n$ is a fundamental discriminant

$$c_k(l^2 n) = c_k(n) \sum_{d|l} \mu(d) \left(\frac{(-1)^k n}{d}\right) d^{k-1} a_{2k}(n/d), \tag{8}$$

for $k > 1$. If $k = 1$, (8) is still true if the summation is taken only over those d's which divide l and are prime to p. In particular,

$$c_1(p^2 n) = c_1(n)$$

for any positive integer n such that $(-1)^k n \equiv 0, 1 \bmod 4$.

Let m and n be positive integers with $(-1)^k m, (-1)^k n \equiv 0, 1 \bmod 4$, and suppose that $(-1)^k n$ is a fundamental discriminant. According to [**6**, Theorem 3] we have

$$c_1(m)c_1(n) = T(1) \sum_{\substack{Q \bmod \Gamma_0(p) \\ |Q| = mn \\ a \equiv 0 \bmod p}} \omega_{-n}(Q) j_Q(f_2) \tag{9}$$

and, since f_{2k} is a modular form on $SL_2(\mathbb{Z})$,

$$c_k(m)c_k(n) = T(k) \sum_{\substack{Q \bmod SL_2(\mathbb{Z}) \\ |Q| = nm}} \omega_{(-1)^k n}(Q) j_Q(f_{2k}) \tag{10}$$

with a non-zero $T(k)$ which does not depend on m and n.

4. Proof of the Theorem. The formulae (9) and (10) do not depend on a particular choice of the systems of representatives in the summations. We formulate some properties of these systems in the following lemmas. Their proofs reduce to direct checks, and we omit them.

Lemma 1 *Assume $\Delta \equiv 0 \bmod p$ Then one can choose a system of representatives $Q \bmod SL_2(\mathbb{Z})$ with $|Q| = \Delta$ such that $a \equiv b \equiv 0 \bmod p$ for any representative $Q = [a, b, c]$*

Lemma 2 *Assume $\Delta \equiv 0 \bmod p$. One can choose a system of representatives $Q \bmod \Gamma_0(p)$ with $|Q| = \Delta$ such that $b \equiv 0 \bmod p$ and $ac \equiv 0 \bmod p$ for any Q in the system.*

Lemma 3 *One can choose the subset of a system of representatives that satisfies the conditions of Lemma 2 and the additional condition $a \equiv 0 \bmod p$ as the system of representatives that satisfies the conditions of Lemma 1.*

In particular, one can choose a common system of representatives for the summations in (9) and (10).

For $\tilde{\kappa} \in \mathcal{X}(\tilde{\Lambda}_1)$, and two discriminants D_0 and D_1 such that D_0 is fundamental, $D_0 D_1 \equiv 0 \bmod p$ with $D_0 D_1 > 0$ put

$$R(D_0, D_1)(\tilde{\kappa}) = \sum_{\substack{Q \bmod SL_2(\mathbb{Z}) \\ |Q| = -D_0 D_1}} \omega_{D_0}(Q) J_Q(\tilde{\kappa}), \tag{11}$$

the system of representatives in the summation is chosen according to Lemmas 2 and 3. Then the proposition in section 2 yields $R(D_0, D_1) \in \tilde{\Lambda}_1$.

Let $-n$ be a negative fundamental discriminant. Put $\tilde{\kappa}_0 = (1, id)$. According to (9), (7), and since the quadratic form Q' runs through a system of representatives if the forms Q do, we obtain

$$R(-n, -mp)(\tilde{\kappa}_0) = \frac{1}{2} \frac{\Omega_{\tilde{\kappa}_0}}{\Omega^-(\tilde{\kappa}_0)} T(1)^{-1} c_1(n) c_1(mp) \tag{12}$$

for a positive integer m with $mp \equiv 0, 3 \bmod 4$.

We also have

$$R(-(-1)^{\frac{p-1}{2}}np, -(-1)^{\frac{p-1}{2}}m)(\xi\tilde{\kappa}_0)$$

$$= \frac{1}{2}\frac{\Omega_{\xi\tilde{\kappa}_0}}{\Omega^-(\xi\tilde{\kappa}_0)} \sum_{\substack{Q \bmod SL_2(\mathbb{Z}) \\ |Q| = nmp}} \omega_{-(-1)^{\frac{p-1}{2}}np}(Q)\xi(c)(j_Q(f_2^*) + j_{Q'}(f_2^*)).$$

Notice that

$$\omega_{-(-1)^{\frac{p-1}{2}}np}(Q)\xi(c) = \omega_{-n}(Q).$$

Thus

$$R(-(-1)^{\frac{p-1}{2}}np, -(-1)^{\frac{p-1}{2}}m)(\xi\tilde{\kappa}_0) = R(-n, -mp)(\tilde{\kappa}_0) \tag{13}$$

Now pick $l \equiv 1 \bmod (p-1)p^{A-1}/2$ with $l \not\equiv 1 \bmod p-1$. We have for $\tilde{\kappa}_1 = (l, id)$

$$R(-(-1)^{\frac{p-1}{2}}np, -(-1)^{\frac{p-1}{2}}m)(\tilde{\kappa}_1) \equiv R(-(-1)^{\frac{p-1}{2}}np, -(-1)^{\frac{p-1}{2}}m)(\xi\tilde{\kappa}_0) \bmod p^A \tag{14}$$

On the other hand, according to (7) and (11),

$$R(-(-1)^{\frac{p-1}{2}}np, -(-1)^{\frac{p-1}{2}}m)(\tilde{\kappa}_1) =$$

$$\frac{1}{2}\frac{\Omega_{\xi\tilde{\kappa}_1}}{\Omega^-(\xi\tilde{\kappa}_1)} \sum_{\substack{Q \bmod SL_2(\mathbb{Z}) \\ |Q| = nmp}} \omega_{-(-1)^{\frac{p-1}{2}}np}(Q)\xi(c)(j_Q(f_{2l}^*) + j_{Q'}(f_{2l}^*)).$$

By Lemma 3 and (2), we obtain

$$j_Q(f_{2l}^*) = j_Q(f_{2l}(\tau) - \beta_{2l}f_{2l}(p\tau)) =$$

$$\int_{C_Q} f_{2l}(\tau)(a\tau^2 + b\tau + c)^{l-1}d\tau - \beta_{2l}\int_{C_Q} f_{2l}(p\tau)(a\tau^2 + b\tau + c)^{l-1}d\tau.$$

We make the variable change $\tau \mapsto \tau/p$ in the second integral. Note that $v_p(\beta_{2l}) = 2l$, and both integrals on the right hand side are $\mathbb{Z}$-linear combinations of periods of the cuspform f_{2l} (Manin's continued fraction trick, [**7**]). We sum over the system of representatives, use (10) for the first integral, and estimate the second one. It follows that

$$R(-(-1)^{\frac{p-1}{2}}np, -(-1)^{\frac{p-1}{2}}m)(\tilde{\kappa}_1) \equiv \frac{1}{2}\frac{\Omega_{\tilde{\kappa}_1}}{\Omega^-(\tilde{\kappa}_1)}c_l(np)c_l(m)T(l)^{-1} \bmod p^l \tag{15}$$

Assume now that we can pick a fundamental discriminant $-n$ such that $n \not\equiv 0 \bmod p$ and $c_1(n) \neq 0$. It follows from (12) that

$$\sum_{m>0} R(-n, -mp)(\tilde{\kappa}_0)q^m = \frac{1}{2}\frac{\Omega_{\tilde{\kappa}_0}}{\Omega^-(\tilde{\kappa}_0)}\frac{c_1(n)}{T(1)}U_p\phi_1.$$

Since $\frac{p-1}{2}p^{A-1} > A$ for $A \geq 1$, the congruences (14) together with (15) yield

$$\sum_{m>0} R(-(-1)^{\frac{p-1}{2}}np, -(-1)^{\frac{p-1}{2}}m)(\xi\tilde{\kappa}_0)q^m \equiv \frac{1}{2}\frac{\Omega_{\tilde{\kappa}_1}}{\Omega^-(\tilde{\kappa}_1)}\frac{c_l(np)}{T(l)}\phi_l \mod p^A$$

The equalities (13) and (12) now yield

$$\frac{\Omega_{\tilde{\kappa}_0}}{\Omega^-(\tilde{\kappa}_0)}\frac{c_1(n)}{T(1)}U_p\phi_1 \equiv \frac{1}{2}\frac{\Omega_{\tilde{\kappa}_1}}{\Omega^-(\tilde{\kappa}_1)}\frac{c_l(np)}{T(l)}\phi_l \bmod p^A, \tag{16}$$

as it was claimed in Theorem.

Our assumption that there exist a fundamental discriminant $-n \not\equiv 0 \bmod p$ such that $c_1(n) \neq 0$, though seems always to be true, is not easy to prove. Anyway, if it were false, one might choose a fundamental discriminant $-mp$ such that $c_1(mp) \neq 0$ (because $\phi_1 \neq 0$). In this case, (12) remains true for arbitrary n. After this point, the argument can be repeated mutatis mutandis.

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