# Math 475 Exercises

**Jan 20, 2010**

(G) **1.** Prove the infinite version of Ramsey's Theorem. That is, show that if every pair of elments $i$ and $j$ in $\omega = \{0, 1, 2, \ldots\}$ is connected by either a red or a blue line, then there is an infinite subset of $\omega$ such that all lines between points in this set have the same color.

**2.** Show that $R(3, 4) > 8$. This says that there is some way of coloring every edge of an 8 point set with either red or blue such that there is no red triangle nor is there a set of four points with all connecting edges blue. Here is the graph that works: take 8 points and arrange them around a circle. For each point color the lines to both its left and right neighbors red and also color the line to the dead opposite point red. All other lines are blue. Show this works.

**3.** I will show in class that $R(3, 4) = 9$. Using this and the fact shown in class:

$$R(m, n) \leq R(m-1, n) + R(m, n-1)$$

prove that $R(3, 5) \leq 14$.

**4.** Prove Lemma 2 on the handout sheet.