# MA 123F – Limit laws with infinite limits

## Fall 2010

Below is a tabular display of the limit laws when infinity is involved. Perhaps the most important thing to notice about this table is all the (?)s. These represent cases where the limit law doesn't tell you anything!! In such cases, more work needs to be done to determine whether the limit exists or not (either rearranging the limit or later techniques such as l'Hospital's Rule).

In the table, $a$ and $b$ are some real numbers, and $f(x)$ and $g(x)$ are two functions whose limit at $a$ is given by the first and second columns, respectively. The third through sixth columns tell you what the limit laws say about the limit as $x$ approaches $a$ of the function in the first row. A '(?)' means that the limit law tells you nothing! A '(!)' means that the limit law tells you the limit does not exist, but it may be $\pm\infty$. Note that you can replace the limits with limits from the left or right.

| $\lim\limits_{x \to a} f(x)$ | $\lim\limits_{x \to a} g(x)$ | $f+g$ | $f-g$ | $fg$ | $f/g$ |
|---|---|---|---|---|---|
| $\infty$ | $\infty$ | $\infty$ | (?) | $\infty$ | (?) |
| | $-\infty$ | (?) | $\infty$ | $-\infty$ | (?) |
| | $0$ | $\infty$ | $\infty$ | (?) | (!) |
| | $\pm b, (\text{with } b > 0)$ | $\infty$ | $\infty$ | $\pm\infty$ | $\pm\infty$ |
| $-\infty$ | $\infty$ | (?) | $-\infty$ | $-\infty$ | (?) |
| | $-\infty$ | $-\infty$ | (?) | $\infty$ | (?) |
| | $0$ | $-\infty$ | $-\infty$ | (?) | (!) |
| | $\pm b, (\text{with } b > 0)$ | $-\infty$ | $-\infty$ | $\mp\infty$ | $\mp\infty$ |
| $0$ | $\infty$ | $\infty$ | $-\infty$ | (?) | $0$ |
| | $-\infty$ | $-\infty$ | $\infty$ | (?) | $0$ |
| | $0$ | $0$ | $0$ | $0$ | (?) |
| | $\pm b, (\text{with } b > 0)$ | $\pm b$ | $\mp b$ | $0$ | $0$ |
| $\pm b, (\text{with } b > 0)$ | $\infty$ | $\infty$ | $-\infty$ | $\pm\infty$ | $0$ |
| | $-\infty$ | $-\infty$ | $\infty$ | $\mp\infty$ | $0$ |
| | $0$ | $\pm b$ | $\pm b$ | $0$ | (!) |

(!): What's happening in this case is that the function is getting bigger and bigger in absolute value. For example, when $f(x) = 1$, $g(x) = x$, and $a = 0$, we're in the last row of the table and

$$\lim_{x \to 0} \frac{1}{x} \text{ does not exist,}$$

but

$$\lim_{x \to 0^+} \frac{1}{x} = \infty \quad \text{and} \quad \lim_{x \to 0^-} \frac{1}{x} = -\infty.$$