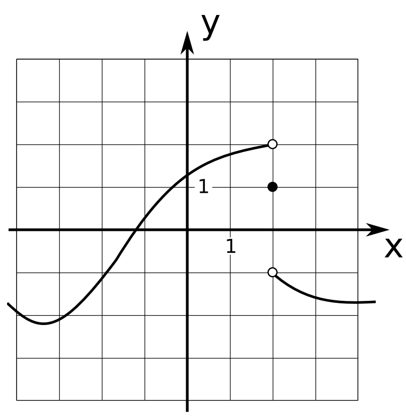
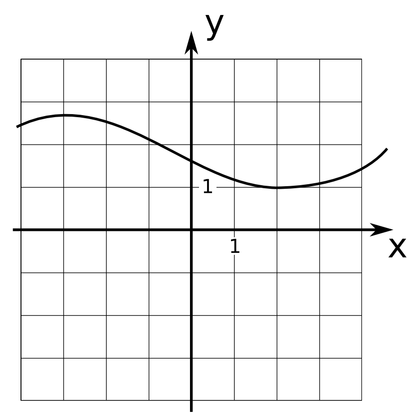
Quiz 1 – MA 123F

Tuesday, Sept. 14, 2010

(1) For this question, use the two graphs below. In each case (a)–(d), determine whether the limit exists, and if so determine its value. Justify your answer by listing the limit laws and other facts you use.



Graph of $f(x)$



Graph of $g(x)$

(a) $\lim_{x\to 2} f(x)$ (b) $\lim_{x\to 2^+} f(x)$ (c) $\lim_{x\to 2} f(x)g(x)$ (d) $\lim_{x\to 2} g(x)$

(2) In each case (a)–(c), determine whether the limit exists, and if so determine its value. Justify your answer by listing the limit laws and other facts you use.

(a) $\lim_{x\to 5} x^2 - 5$ (b) $\lim_{x\to 0} x^2 + \dfrac{1}{x-1}$ (c) $\lim_{x\to 1} \dfrac{x^2+5}{x-1}$

Solutions to Quiz 1

MA 123F

(1) (a) The graph shows that $\lim\limits_{x\to 2^-} f(x) = 2$ & $\lim\limits_{x\to 2^+} f(x) = -1$

Since these are not equal, $\boxed{\lim\limits_{x\to 2} f(x) \text{ Does Not Exist}}$

(b) By following the graph from the right, we see that $\boxed{\lim\limits_{x\to 2^+} f(x) = -1}$

(c) Since $\lim\limits_{x\to 2^+} f(x)$ & $\lim\limits_{x\to 2^+} g(x)$ both exist

the product law says that
$$\lim_{x\to 2^+} f(x)g(x) = \left(\lim_{x\to 2^+} f(x)\right)\cdot\left(\lim_{x\to 2^+} g(x)\right)$$
$$= (-1)\cdot(2) = -2$$

Similarly, $\lim\limits_{x\to 2^-} f(x) = 2$ & $\lim\limits_{x\to 2^-} g(x) = 1$

so the product law says that $\lim\limits_{x\to 2^-} f(x)g(x) = 2\cdot 1 = 2$

Since $\lim\limits_{x\to 2^-} f(x)g(x) = 2 \neq -2 = \lim\limits_{x\to 2^+} f(x)g(x)$, the $\boxed{\lim\limits_{x\to 2} f(x)g(x) \text{ Does Not Exist}}$

(d) By following the function from the left & from the right, we see that $\boxed{\lim\limits_{x\to 2} g(x) = 1}$

(2) (a) $f(x) = x^2 - 5$ is a polynomial & the Direct Substitution Property says that $\lim\limits_{x\to 5} f(x) = f(5) = 5^2 - 5 = \boxed{20}$

(b) Try using the sum law: x^2 is a polynomial so the Direct Substitution Property says that $\lim\limits_{x\to 0} x^2 = 0^2 = 0$.

$\frac{1}{x-1}$ is a rational function & 0 is in its domain so the Direct Substitution Property says that $\lim\limits_{x\to 0} \frac{1}{x-1} = \frac{1}{0-1} = -1$. Since $\lim\limits_{x\to 0} x^2$ & $\lim\limits_{x\to 0} \frac{1}{x-1}$ both exist, the sum law says $\lim\limits_{x\to 0} x^2 + \frac{1}{x-1} = (0) + (-1) = \boxed{-1}$

(c) $f(x) = \frac{x^2+5}{x-1}$ is a rational function, but $x=1$ is not in its domain. Still, if we plug in $x=1$, we get $\frac{1^2+5}{1-1} = \frac{6}{0}$. This is a non-zero number over 0, so the $\boxed{\text{limit does not exist.}}$