Quiz 2 – MA 123F – Tuesday, Sept. 21, 2010

Name:

Note: Question 2 is on the other side of this sheet.

(1) In each case (a)–(c), determine whether the limit exists, and if so determine its value. Show your work.

(a) $\lim_{x \to 1} \frac{1}{x-1} - \frac{1}{x(x-1)}$

(b) $\lim_{x \to -1^+} \frac{\sqrt{x+1}}{\sqrt{x+5}-2}$

(c) $\lim_{x \to 0} f(x)$, where $f(x) = \begin{cases} e^x \cos(x) & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

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(2) In each case (a)–(b), determine where the function $f(x)$ is discontinuous. Justify your answer.

(a) $f(x) = e^x \sin(x) - \dfrac{1}{x}$

(b) $f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 1 & \text{if } x = 0 \\ \frac{\sin(x)}{\ln(x)} & \text{if } x > 0 \end{cases}$

Solutions to Quiz 2 - MA123F

(1) (a) $\frac{1}{x-1} - \frac{1}{x(x-1)} = \frac{\cancel{x-1}}{x\cancel{(x-1)}} = \frac{1}{x}$ for $x \neq 0, 1$

so $\lim_{x\to 1} \frac{1}{x-1} - \frac{1}{x(x-1)} = \lim_{x\to 1} \frac{1}{x} = \boxed{1}$

↳ Direct Substitution

(b) $\frac{\sqrt{x+1}}{\sqrt{x+5}-2} \cdot \frac{(\sqrt{x+5}+2)}{(\sqrt{x+5}+2)} = \frac{\sqrt{x+1}(\sqrt{x+5}+2)}{x+5-4} = \frac{\sqrt{x+1}(\sqrt{x+5}+2)}{x+1}$ for $x > -1$

so $\lim_{x\to -1^+} \frac{\sqrt{x+1}}{\sqrt{x+5}-2} = \lim_{x\to -1} \frac{\sqrt{x+5}+2}{\sqrt{x+1}}$. Plugging in gives $\frac{4}{0}$ $(4 \neq 0)$

so limit $\boxed{\text{DNE}}$

(c) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} e^x \cos x = e^0 \cos 0 = 1$

↳ since e^x & $\cos x$ are continuous, so $e^x \cos x$ is, too.

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x = 0 \neq 1$, so limit $\boxed{\text{DNE}}$

(2)(a) e^x & $\sin x$ are continuous everywhere, so by the product law, so is $e^x \sin x$.

$\frac{1}{x}$ is continuous for $x \neq 0$. By the difference law, $e^x \sin x - \frac{1}{x}$ is

$\boxed{\text{continuous for all } x \neq 0.}$

(b) If $x < 0$, $f(x) = x^2$, so $f(x)$ is continuous for all $x < 0$

If $x > 0$, $f(x) = \frac{\sin x}{\ln x}$; $\sin x$ is continuous everywhere & $\ln x$ is continuous for all $x > 0$, but $\ln(1) = 0$, so the quotient law says that $f(x)$ is continuous for all $x > 0$ except possibly $x = 1$

At $x=1$, $\lim_{x\to 1} \frac{\sin x}{\ln x}$ DNE since plugging in gives $\frac{\sin(1) \neq 0}{0}$

so $f(x)$ is not continuous at $x = 1$ (it's also not defined there).

At $x=0$: $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x^2 = 0$, but $f(0) = 1 \neq 0$, so $f(x)$ is not continuous at $x = 0$.

~~$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sin x}{\ln x}$~~

So $\boxed{f(x) \text{ is continuous on } \{x \neq 0, 1\}.}$