Quiz 3 – MA 123F – Tuesday, Sept. 28, 2010

Name:

(1) In each case (a)–(b), determine the function's vertical and horizontal asymptotes. Show your work.

(a) $f(x) = \dfrac{2x^2+1}{(x-1)(x-3)}$

(b) $f(x) = x + \dfrac{1}{(x-1)^2}$

(2) Let $g(x) = x(x-1)$ and let

$$h(x) = \begin{cases} x+1, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0. \end{cases}$$

Show that $f(x) = g(h(x))$ is continuous everywhere. Justify your answer.

Solutions to Quiz 3 – MA123F

(1)(a) Vertical asymptotes: By quotient rule, $f(x)$ is continuous on $\{x \neq 1, 3\}$, so only possible vertical asymptotes are $x=1$ & $x=3$.

$x=1$: $\lim_{x\to 1^-} f(x) = -\infty$ so $\boxed{x=1 \text{ is a vertical asymptote}}$

$x=3$: $\lim_{x\to 3^-} f(x) = -\infty$ so $\boxed{x=3 \text{ is a vertical asymptote}}$

Horizontal asymptotes: Check $\lim_{x\to\infty}$ & $\lim_{x\to-\infty}$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^2\left(2+\frac{1}{x^2}\right)}{x^2-4x+3} = \lim_{x\to\infty} \frac{\cancel{x^2}\left(2+\frac{1}{x^2}\right)}{\cancel{x^2}\left(1-\frac{4}{x}+\frac{3}{x^2}\right)}$$

$\lim_{x\to\infty} 2+\frac{1}{x^2} = 2$ $\lim_{x\to\infty} 1-\frac{4}{x}+\frac{3}{x^2} = 1$

so $= \frac{2}{1} = 2$ so $\boxed{y=2 \text{ is a horiz. asymptote}}$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{\left(2+\frac{1}{x^2}\right)}{\left(1-4/x-3/x^2\right)}$$

$\lim_{x\to-\infty} 2+\frac{1}{x^2} = 2$ $\lim_{x\to-\infty} 1-4/x-3/x^2 = 1$

so $= \frac{2}{1} = 2$ so no new horizontal asymptote

(b) Vert. asymptotes: By quot. rule, $f(x)$ is cont. on $\{x\neq 1\}$, so only possible vert. asymptote is $x=1$.

$x=1$: $\lim_{x\to 1} x + \frac{1}{(x-1)^2} = 1+\infty = \infty$. so $\boxed{x=1 \text{ is a vert. asymptote}}$

Horiz. asymptotes: $\lim_{x\to\infty} f(x) = \infty + \frac{1}{\infty} = \infty + 0 = \infty$ no horiz. asymptote from $x\to\infty$

$\lim_{x\to-\infty} f(x) = -\infty + \frac{1}{\infty} = -\infty$ so $\boxed{\text{no horiz. asymptotes}}$

(2) $g(x)$ is continuous everywhere. $h(x)$ is cont. for $x>0$ so $\boxed{g(h(x)) \text{ is cont. for } x>0}$

$\boxed{h(x) \text{ is cont. for } x<0,}$ so $g(h(x))$ is cont for $x<0$. Only problem may occur at $x=0$. Check definition: $\lim_{x\to 0^-} g(h(x)) = \lim_{x\to 0^-} g(-x) = \lim_{x\to 0^-} x(-x-1) = 0\cdot(0-1) = \boxed{0}$

$\lim_{x\to 0^+} g(h(x)) = \lim_{x\to 0^+} g(x+1) = \lim_{x\to 0^+} (x+1)(x+\cancel{1}-\cancel{1}) = 1\cdot 0 = \boxed{0}$

& $g(h(0)) = g(1) = 1\cdot(1-1) = \boxed{0}$

all equal, so $\boxed{g(h(x)) \text{ cont. at } x=0}$