Quiz 5 – MA 123F – Tuesday, Oct. 26, 2010

Name:

Show your work.

(1) Find $\dfrac{d}{dx}\Big((\sin(x))^2 + \tan(x)\Big)$.

(2) If $\cos(xy) = x$, find y'.

(3) If $e^{xy} = 1$, find y'' at the point $(2, 0)$.

MA123F - Solutions to Quiz 5

(1) $\frac{d}{dx}(\sin^2 x + \tan x) = 2\sin x \frac{d}{dx}\sin x + \sec^2 x = \boxed{2\sin x\cos x + \sec^2 x}$

$(= \sin 2x + \sec^2 x)$

(2) [Take $\frac{d}{dx}$ of both sides & solve for y'.]

$$\frac{d}{dx}\left(\cos(xy)\right) = \frac{d}{dx}(x)$$

$$-\sin(xy)\cdot\frac{d}{dx}(xy) = 1$$

$$-\sin(xy)(xy'+y) = 1$$

so $-xy'\sin(xy) = 1 + y\sin(xy)$

so $\boxed{y' = \dfrac{-1-y\sin(xy)}{x\sin(xy)}}$

(3) $\frac{d}{dx}(e^{xy}) = \frac{d}{dx}(1)$

$$e^{xy}\frac{d}{dx}(xy) = 0$$

$$e^{xy}(xy'+y) = 0$$

so $xy'\cancel{e^{xy}} = -y\cancel{e^{xy}}$

so $y' = \frac{-y}{x}$

so $y'' = \frac{x\cdot(-y') - -y\cdot 1}{x^2}$

$y'' = \frac{y - xy'}{x^2}$

[Since we just want the value of y'' at $(2,0)$, we can find the value of y' at $(2,0)$ & plug this into the formula for y'' & get the answer.]

so, at $(2,0)$, $y' = \frac{-0}{2} = 0$. so, at $(2,0)$, $y'' = \frac{0-2\cdot 0}{2^2} = \boxed{0}$

[If you were asked for a formula for y'', you need to substitute the formula $y' = \frac{-y}{x}$ into the formula for y''. You would get

$y'' = \frac{y - x\cdot(\frac{-y}{x})}{x^2}$ so $\boxed{y'' = \frac{2y}{x^2}}$.]