Quiz 6 – MA 123F – Tuesday, Nov. 2, 2010

Name:

Show your work.

(1) What is $\arcsin(1/2)$?

(2) Find $\dfrac{d}{dx}\left(\sqrt{\arccos(x)}\right)$.

(3) Let $f(x) = \arcsin(x^3 + 1)$.

(a) What is the domain of $f(x)$?

(b) What is the range of $f(x)$?

(c) What is $f'(x)$?

MA123F - Solutions to Quiz 6

(1) [By definition, $\arcsin(1/2)$ satisfies (i) $\sin(\arcsin(1/2)) = \frac{1}{2}$
(ii) $-\pi/2 \le \arcsin(1/2) \le \pi/2$.

These two conditions are always enough to give the value of $\arcsin(1/2)$.]

sin of what is $1/2$? $\sin(\pi/6) = \frac{1}{2}$ & $-\pi/6 \le \pi/6 \le \pi/2$ so

$$\boxed{\arcsin(1/2) = \pi/6}$$

(2) $$\frac{d}{dx}\left(\sqrt{\arccos(x)}\right) = \frac{1}{2\sqrt{\arccos(x)}} \frac{d}{dx}(\arccos(x)) = \frac{1}{2\sqrt{\arccos(x)}} \cdot \frac{-1}{\sqrt{1-x^2}}$$
$$= \boxed{\frac{-1}{2\sqrt{1-x^2}\sqrt{\arccos(x)}}}$$

(3) (a) domain of x^3+1 is $(-\infty, \infty)$ (so no restrictions on x are imposed by x^3+1).
domain of $\arcsin(x)$ is $[-1, 1]$, x must satisfy $-1 \le x^3+1 \le 1$

(i) $-1 \le x^3+1$ (ii) $x^3+1 \le 1$

so $-2 \le x^3$ so $-\sqrt[3]{2} \le x$ so $x^3 \le 0$ so $x \le 0$

so $\boxed{\text{domain of } f(x) \text{ is } -\sqrt[3]{2} \le x \le 0}$

(b) Since the values of x^3+1 fill up the domain of $\arcsin(x)$,

the range of $\arcsin(x^3+1)$ = range of $\arcsin(x)$

so range of $\arcsin(x^3+1)$ is $\boxed{[-\pi/2, \pi/2]}$

(c) $$f'(x) = \frac{1}{\sqrt{1-(x^3+1)^2}} \cdot \frac{d}{dx}(x^3+1) = \boxed{\frac{3x^2}{\sqrt{1-(x^3+1)^2}}}$$