**Quiz 7 – MA 123F – Tuesday, Nov. 9, 2010**

Name:

**Show your work.**

(1) Differentiate $\log_2\left(\dfrac{1+x}{1-x}\right)$.

(2) Differentiate $x^{x^2}$.

(3) Use the linear approximation of $f(x) = \ln(1+2x)$ at $x = 0$ to estimate $\ln(1.02)$.

# MA 123F - Solutions to Quiz 7

(1) $\log_2\left(\frac{1+x}{1-x}\right) = \log_2(1+x) - \log_2(1-x)$

so $\frac{d}{dx}\left( \quad '' \quad \right) = \frac{1}{(1+x)\ln(2)} - \frac{1}{(1-x)\ln(2)}\cdot(-1) = \boxed{\frac{1}{\ln(2)}\left(\frac{1}{1+x} + \frac{1}{1-x}\right)}$

$= \frac{1}{\ln(2)}\left(\frac{1-\cancel{x}+1+\cancel{x}}{1-x^2}\right) = \frac{2}{\ln(2)(1-x^2)}$

(2) $y = x^{x^2}$ Then $\ln(y) = \ln(x^{x^2}) = x^2\ln(x)$

so $\frac{1}{y}y' = x^{\cancel{2}}\cdot\frac{1}{\cancel{x}} + \ln(x)\cdot 2x = x\left(1+2\ln(x)\right)$

so $y' = yx(1+2\ln(x)) = x^{x^2}x(1+2\ln(x)) = \boxed{x^{(x^2+1)}(1+2\ln(x))}$

(3) $f(x) = \ln(1+2x)$. Near $x=0$, $f(x) \approx f(0) + f'(0)(x-0)$

$f(0) = \ln(1) = 0$, $f'(x) = \frac{1}{1+2x}\cdot 2$

so $f'(0) = \frac{2}{1+0} = 2$

so $\underline{f(x) \approx 2x}$

$1.02 = 1 + 2\cdot 0.01$ so $\ln(1.02) \approx 2\cdot 0.01$

$\approx \boxed{0.02}$