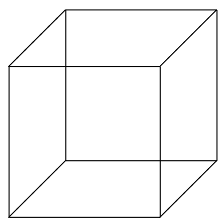
Quiz 8 – MA 123F – Tuesday, Nov. 16, 2010

Name:

Show your work.

(1) The pictured tank is shaped like a cube of side 2 m. It is being filled with water at a rate of 2 m^3/min. At what rate is the height of the water changing?



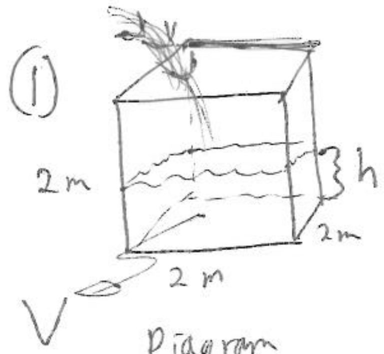
(2) Let $f(x) = \dfrac{x-1}{x^2+3}$.

(a) Find all critical points of $f(x)$.

(b) Find the absolute maximum and the absolute minimum of $f(x)$ on the interval $[-2, 2]$.

MA123F - Solutions to Quiz 8

(1)



Diagram

Know: $\frac{dV}{dt} = 2\,m^3/min$

Question: $\frac{dh}{dt} = ?$

Relation: $V = (2m)\cdot(2m)\cdot h = 4h\ m^2$

so $\frac{dV}{dt} = (4\ m^2)\,\frac{dh}{dt}$

so $\frac{dh}{dt} = \frac{1}{4m^2}\,\frac{dV}{dt} = \frac{1}{4\cancel{m^2}}\,2\,m^{\cancel{3}}/min$

so $\boxed{\frac{dh}{dt} = \frac{1}{2}\ m/min}$

(Sanity check: It makes sense that the height is increasing (i.e. $\frac{dh}{dt} > 0$).)

(2) (a) $f'(x) = \frac{(x^2+3)\cdot 1 - (x-1)\cdot 2x}{(x^2+3)^2} = \frac{x^2+3-2x^2+2x}{(x^2+3)^2} = \frac{-(x^2-2x-3)}{(x^2+3)^2}$

(i) $f'(x) = 0$: when $(x^2-2x-3) = 0$
$= (x-3)(x+1)$
so $\boxed{x=+3}$ & $\boxed{x=-1}$

(ii) $f'(x)$ undefined: denominator is never 0, so f(x) defined everywhere

so $\boxed{\text{critical points are } x=-1\ \&\ x=3}$

(b) Table:

x	f(x)	
−1	$\frac{-2}{4} = \frac{-1}{2}$	→ min
−2	$\frac{-2-1}{7} = \frac{-3}{7}$	
2	$\frac{2-1}{7} = \frac{1}{7}$	→ max

[Note: x=3 is not in the table because it is not in [-2,2].]

so on [-2,2]:

$$\boxed{\begin{array}{l}\text{abs. max at } x=2 \quad (f(2)=\tfrac{1}{7}) \\ \text{abs. min. at } x=-1 \quad (f(-1)=\tfrac{-1}{2})\end{array}}$$