

Quiz 9 – MA 123F – Tuesday, Nov. 23, 2010

Name:

Show your work.

1. Find the local maxima and minima of $f(x) = x^2\sqrt{x+3}$ for $x > -3$.

2. In each case (a), (b) determine whether the limit exists, and if so, determine its value. Show your work.

(a) $\lim_{x \rightarrow 0} \frac{\ln(1+x) - x}{x}$

(b) $\lim_{x \rightarrow \infty} e^{-x} \ln(x)$

MA123F - Solutions to Quiz 9

1) Critical points: $f'(x) = x^2 \frac{1}{2\sqrt{x+3}} + 2x\sqrt{x+3} = \frac{x^2 + 4x(x+3)}{2\sqrt{x+3}} = \frac{x(5x+12)}{2\sqrt{x+3}}$

(i) $f'(x)=0$: when $x=0$ or $x = -\frac{12}{5}$

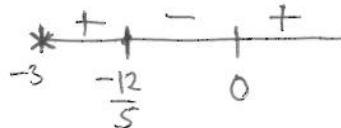
(ii) $f'(x)$ undefined: $f'(x)$ is defined everywhere for $x > -3$

[At this point you have two options: option (A): The First Derivative test
let's do both.]
option (B): The second Derivative test

Option (A): First deriv. test: $f'(x)$

$$f'(x) = \frac{x(5x+12)}{2\sqrt{x+3}}$$

$\rightarrow > 0$ for all $x > -3$



so

$x=0$ is a local min
 $x = -\frac{12}{5}$ is a local max

> 0 if $x < -\frac{12}{5}$
 < 0 if $-\frac{12}{5} < x < 0$
 > 0 if $x > 0$

Option (B): Second deriv. test:

$$f''(x) = \frac{2\sqrt{x+3}(10x+12) - x(5x+12) \cdot \frac{1}{\sqrt{x+3}}}{4(x+3)}$$

so $f''(0) = \frac{2\sqrt{3} \cdot 12 - 0}{4 \cdot 3} > 0$ so $x=0$ is a local min

$\Delta f''(-\frac{12}{5}) = \frac{2\sqrt{\frac{3}{5}} \cdot (-12) - 0}{4 \cdot \frac{3}{5}} < 0$ so $x = -\frac{12}{5}$ is a local max

(2)(a) $\lim_{x \rightarrow 0} \frac{0-0}{0} = \frac{0}{0}$. use L'Hospital's rule, so $= \lim_{x \rightarrow 0} \frac{\frac{1}{1+x} - 1}{1} = \frac{1-1}{1} = \boxed{0}$

(b) $\lim_{x \rightarrow \infty} 0 \cdot \infty$. rewrite, so $= \lim_{x \rightarrow \infty} \frac{\ln(x)}{e^x} = \frac{\infty}{\infty}$. L'Hospital's rule $= \lim_{x \rightarrow \infty} \frac{(\frac{1}{x})}{e^x} = \lim_{x \rightarrow \infty} \frac{1}{x e^x}$
 $= \frac{1}{\infty \cdot \infty}$
 $= \frac{1}{\infty}$
 $= \boxed{0}$