Quiz 9 – MA 123F – Tuesday, Nov. 23, 2010

Name:

Show your work.

1. Find the local maxima and minima of $f(x) = x^2\sqrt{x+3}$ for $x > -3$.

2. In each case (a), (b) determine whether the limit exists, and if so, determine its value. Show your work.

   (a) $\lim_{x \to 0} \frac{\ln(1+x) - x}{x}$

   (b) $\lim_{x \to \infty} e^{-x} \ln(x)$

MA123F – Solutions to Quiz 9

1) Critical points: $f'(x) = x^2 \dfrac{1}{2\sqrt{x+3}} + 2x\sqrt{x+3} = \dfrac{x^2 + 4x(x+3)}{2\sqrt{x+3}} = \dfrac{x(5x+12)}{2\sqrt{x+3}}$

(i) $f'(x) = 0$: when $\boxed{x=0}$ or $\boxed{x = -\frac{12}{5}}$

(ii) $f'(x)$ undefined: $f'(x)$ is defined everywhere for $x > -3$

[At this point you have two options: option Ⓐ: The First Derivative test
Let's do both.] option Ⓑ: The second Derivative test

Option Ⓐ: First deriv. test:

$f'(x) = \dfrac{x(5x+12)}{2\sqrt{x+3}}$

$2\sqrt{x+3} > 0$ for all $x > -3$

$x(5x+12)$: > 0 if $x < -\frac{12}{5}$; < 0 if $-\frac{12}{5} < x < 0$; > 0 if $x > 0$

$f'(x)$: $+$ on $(-3, -\frac{12}{5})$, $-$ on $(-\frac{12}{5}, 0)$, $+$ for $x > 0$

so $\boxed{\begin{array}{l} x=0 \text{ is a local min} \\ x = -\frac{12}{5} \text{ is a local max} \end{array}}$

Option Ⓑ: Second deriv. test:

$$f''(x) = \frac{2\sqrt{x+3}\,(10x+12) - x(5x+12)\cdot\frac{1}{\sqrt{x+3}}}{4(x+3)}$$

so $f''(0) = \dfrac{2\cdot\sqrt{3}\cdot 12 - 0}{4\cdot 3} > 0$ so $\boxed{x=0 \text{ is a local min}}$

$f''\left(-\frac{12}{5}\right) = \dfrac{2\sqrt{3/5}\cdot(-12) - 0}{4\cdot 3/5} < 0$ so $\boxed{x = \frac{-12}{5} \text{ is a local max}}$

(2)(a) get $= \dfrac{0-0}{0} = \dfrac{0}{0}$. use L'Hospital's rule, so $= \lim\limits_{x\to 0} \dfrac{\frac{1}{1+x} - 1}{1} = \dfrac{1-1}{1} = \boxed{0}$

(b) get $= 0\cdot\infty$. rewrite, so $= \lim\limits_{x\to\infty} \dfrac{\ln(x)}{e^x} = \dfrac{\infty}{\infty} \xrightarrow{\text{L'Hosptial's rule}} \lim\limits_{x\to\infty} \dfrac{\left(\frac{1}{x}\right)}{e^x} = \lim\limits_{x\to\infty} \dfrac{1}{xe^x}$

$= \dfrac{1}{\infty\cdot\infty}$

$= \dfrac{1}{\infty}$

$= \boxed{0}$