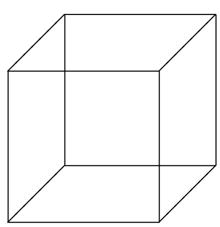
A compendium of quizzes – MA 123F – Fall 2010

Comments: Collected here are all the problems that appeared on your quizzes this semester, listed in somewhat random order. The intention of the quizzes was to make sure you were keeping up with the material, consequently the questions are mostly straightforward, i.e. there will be questions on the final that will be harder than all of the questions below. Still, going through these quiz questions will give you a diagnostic test on whether you can do most of the basic things that will be expected of you on the final. Remember that all the solutions are available on the course website. If there are problems that you can't do, or don't think you would be able to do on a final, come see me in my extended office hours and we can clarify what you don't understand (at the very least send me an email and I can try to explain something). One last caveat I want to mention is that because of the class schedule and time constraints not all the material we covered ended up on a quiz, so there is some basic knowledge that you might be missing that won't be detected by doing these problems.

Show your work.

(1) The pictured tank is shaped like a cube of side 2 m. It is being filled with water at a rate of 2 m^3/min. At what rate is the height of the water changing?



(2) In each case (a)–(b), evaluate the integral.

(a) $\displaystyle\int_1^2 \left(6z^2 - \frac{1}{z^2}\right) dz$

(b) $\displaystyle\int_1^{\pi/4} \left(\frac{1}{x} - \sec^2(x)\right) dx$

(3) In each case (a)–(b), determine the function's vertical and horizontal asymptotes. Show your work.

(a) $f(x) = \dfrac{2x^2+1}{(x-1)(x-3)}$

(b) $f(x) = x + \dfrac{1}{(x-1)^2}$

(4) Find the local maxima and minima of $f(x) = x^2\sqrt{x+3}$ for $x > -3$.

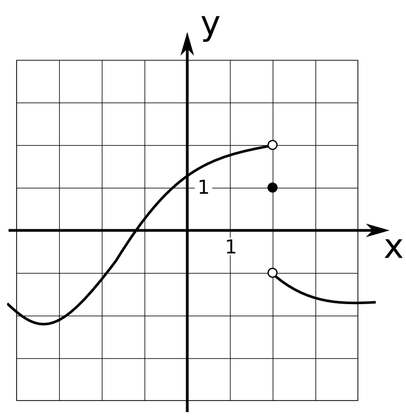
(5) Let $f(x) = \dfrac{x-1}{x^2+3}$.

(a) Find all critical points of $f(x)$.

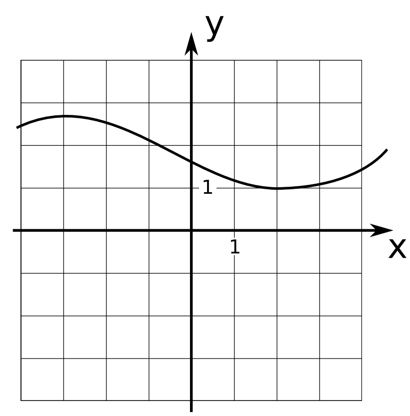
(b) Find the absolute maximum and the absolute minimum of $f(x)$ on the interval $[-2, 2]$.

(6) Use the linear approximation of $f(x) = \ln(1+2x)$ at $x = 0$ to estimate $\ln(1.02)$.

(7) For this question, use the two graphs below. In each case (a)–(d), determine whether the limit exists, and if so determine its value.



Graph of $f(x)$



Graph of $g(x)$

(a) $\lim_{x\to 2} f(x)$ (b) $\lim_{x\to 2^+} f(x)$ (c) $\lim_{x\to 2} f(x)g(x)$ (d) $\lim_{x\to 2} g(x)$

(8) What is $\arcsin(1/2)$?

(9) In each case (a)–(b), give the most general antiderivative of the given function.

(a) $f(x) = \dfrac{3\sqrt{x} - x}{x^2} + e^x$

(b) $g(t) = \dfrac{\sin(t)}{\cos^2(t)}$

(10) Find $\dfrac{d}{dx}\left(\sqrt{\arccos(x)}\right)$.

(11) Let $f(x) = \dfrac{e^x}{(1+e^x)}$. Find $f'(x)$ and $f'(0)$.

(12) Differentiate x^{x^2}.

(13) Find $\dfrac{d}{dx}\left(\dfrac{x^{3/2}}{e^x}\right)$.

(14) If $\cos(xy) = x$, find y'.

(15) Let $y = \dfrac{5e^x}{x}$. Find y' and y''.

(16) Find $\dfrac{d}{dx}\big((\sin(x))^2 + \tan(x)\big)$.

(17) Let $f(x) = \arcsin(x^3 + 1)$.

(a) What is the domain of $f(x)$?

(b) What is the range of $f(x)$?

(c) What is $f'(x)$?

(18) If $e^{xy} = 1$, find y'' at the point $(2, 0)$.

(19) Differentiate $\log_2\left(\dfrac{1+x}{1-x}\right)$.

(20) Find $\dfrac{d}{dx}\left(\left(6\sqrt{x} - \dfrac{3}{\sqrt{x}}\right)e^x\right)$.

(21) In each case (a), (b) determine whether the limit exists, and if so, determine its value. Show your work.

(a) $\displaystyle\lim_{x\to 0} \frac{\ln(1+x) - x}{x}$

(b) $\displaystyle\lim_{x\to\infty} e^{-x}\ln(x)$

(22) In each case (a)–(c), determine whether the limit exists, and if so determine its value. Justify your answer by listing the limit laws and other facts you use.

(a) $\displaystyle\lim_{x\to 5} x^2 - 5$ (b) $\displaystyle\lim_{x\to 0} x^2 + \frac{1}{x-1}$ (c) $\displaystyle\lim_{x\to 1} \frac{x^2+5}{x-1}$

(23) In each case (a)–(c), determine whether the limit exists, and if so determine its value. Show your work.

(a) $\displaystyle\lim_{x\to 1} \frac{1}{x-1} - \frac{1}{x(x-1)}$

(b) $\displaystyle\lim_{x\to -1^+} \frac{\sqrt{x+1}}{\sqrt{x+5} - 2}$

(c) $\displaystyle\lim_{x\to 0} f(x)$, where $f(x) = \begin{cases} e^x\cos(x) & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

(24) Let $g(x) = x(x-1)$ and let

$$h(x) = \begin{cases} x+1, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0. \end{cases}$$

Show that $f(x) = g(h(x))$ is continuous everywhere. Justify your answer.

(25) In each case (a)–(b), determine where the function $f(x)$ is discontinuous. Justify your answer.

(a) $f(x) = e^x \sin(x) - \dfrac{1}{x}$

(b) $f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 1 & \text{if } x = 0 \\ \frac{\sin(x)}{\ln(x)} & \text{if } x > 0 \end{cases}$