Assignment 1 – MA 542

Due in class: Wednesday, Jan. 20, 2010

Chapter 12: 8, 20, 23, 24, 28

(1) Let R be a commutative ring with unity. Show that $a \in R$ is a unit if, and only if, there exists $b \in R$ such that ab is a unit.

(2) (a) An element x of a ring R is called *nilpotent* if there exists a positive integer n such that $x^n = 0$. Let R be a commutative ring with identity. Show that if $x \in R$ is nilpotent, then $1 - x$ is a unit.

(b) Deduce that if R is a commutative ring with identity, u is a unit, and x is nilpotent, then $u - x$ is a unit.

(3) Let X be a set and R a ring. Let $S = \{\text{functions from } X \text{ to } R\}$. Define addition and multiplication on S as follows:

$$(f + g)(x) = f(x) + g(x)$$

and

$$(fg)(x) = f(x)g(x).$$

Show that S is a ring.